

# **Time Series Analysis by Higher Order Crossings**

**Benjamin Kedem**

Department of Mathematics and Institute for Systems Research  
University of Maryland, College Park



The Institute of Electrical and Electronics Engineers, Inc., New York

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Problems and Complements	18
<b>2</b>	<b>Fundamentals of Probability Theory</b>	<b>23</b>
2.1	Random Variables	23
2.2	The Distribution Function	24
2.2.1	Some Order Relations	27
2.3	Examples of Distributions	28
2.4	Multivariate Concepts	36
2.4.1	Moments and Correlation	39
2.4.2	The Correlation Coefficient	40
2.4.3	The Joint Moment-Generating Function	43
2.4.4	The Bivariate Normal Distribution	44
2.4.5	An Orthant Probability	47
2.4.6	The Multivariate Normal Distribution	51
2.5	Modes of Convergence	53
2.6	Problems and Complements	56
<b>3</b>	<b>Elements of Stationary Processes</b>	<b>61</b>
3.1	The Notion of Stationarity	62

3.1.1	Types of Oscillation	62
3.1.2	Definition of a Stationary Process	64
3.1.3	Complex-Valued Stationary Processes	68
3.1.4	Examples of Stationary Processes	72
3.2	Representation of Stationary Processes	82
3.2.1	A Stochastic Integral	82
3.2.2	Spectral Representation	84
3.3	Decomposition of the Spectral Distribution	86
3.4	Applications of the Spectral Representation	89
3.4.1	Stationary Solutions of Stochastic Difference Equations	89
3.4.2	Effect of a Linear Filter	92
3.4.3	Mean Square Ergodicity	101
3.5	Appendix to Chapter 3	103
3.6	Problems and Complements	105
<b>4</b>	<b>Zero-Crossings and Autocorrelation</b>	<b>115</b>
4.1	Zero-Crossings in Discrete Time	116
4.2	The Cosine Formula	118
4.2.1	The Case of a Pure Sinusoid	118
4.2.2	The General Ellipsoidal Case	120
4.2.3	The Mixed Spectrum Case	122
4.2.4	Explosive Oscillation	125
4.3	Transformed Gaussian Processes	126
4.3.1	The Discrete Time Case	126
4.3.2	Extension to Continuous Time	129
4.3.3	Level-Crossing Rate	132
4.4	Effect of Monotone Gain	133
4.5	Problems and Complements	136
<b>5</b>	<b>Higher Order Crossings and Correlations</b>	<b>144</b>
5.1	Parametric Families of Zero-Crossing Counts	145
5.1.1	The Ideal Bandpass Case	147
5.2	Higher Order Correlations	148
5.3	Properties of HOC from Differences	150
5.4	HOC from Repeated Summation	161
5.4.1	Discrete Spectrum Analysis by HOC	162
5.5	The Slutsky Filter	164
5.6	HOC from the $AR(1)$ Filter	165
5.7	Appendix to Chapter 5	171
5.8	Problems and Complements	172

<b>6</b>	<b>Statistical Properties of HOC</b>	<b>177</b>
6.1	Asymptotic Normality	178
6.2	Variance Approximation	179
6.2.1	A Markov Approximation	179
6.2.2	The Case of $m$ -Dependence	182
6.2.3	A White Noise Test	185
6.3	Asymptotic Mixed Spectrum Results	187
6.3.1	The Discrete Spectrum of $d_t$	188
6.3.2	Case of a Sinusoid Plus Noise	190
6.3.3	Zero-Crossing Rate of Band-Limited Processes	191
6.3.4	Model of Random Phases	194
6.4	Consistency of the Sample Autocorrelation	195
6.5	Appendix to Chapter 6	197
6.6	Problems and Complements	200
<b>7</b>	<b>Frequency Estimation by HOC</b>	<b>206</b>
7.1	Introduction and Overview	206
7.2	HOC and Periodogram	209
7.3	Contraction Mappings in Frequency Estimation	215
7.3.1	Detection of a Single Frequency in Noise: The HK Algorithm	216
7.3.2	An Extension: The CM Algorithm	220
7.3.3	Contractions from Band-Pass Filters	223
7.3.4	Two Parametric Families of Filters	226
7.3.5	Stochastic Convergence of the HK and CM Algorithms	231
7.3.6	Examples	238
7.4	A Multivariate CM Method	244
7.4.1	Extending the HOC Idea: Parametrized Least Squares	244
7.4.2	The AR Method and its Unbiased Modification	245
7.5	Summary	251
7.6	Problems and Complements	252
<b>8</b>	<b>Signal Discrimination by HOC</b>	<b>260</b>
8.1	The $\psi^2$ Statistic	261
8.1.1	$\psi^2$ as a Measure of Distance from White Noise	266
8.1.2	Weighted $\psi^2$	267
8.2	HOC Plots	268
8.2.1	Simple HOC Plots	268
8.2.2	HOC Plots from the $\alpha$ -Filter	270
8.2.3	Power Simulation	272
8.3	Scatter Plots	273

8.4	Online Tracking	275
8.5	Problems and Complements	280
<b>9</b>	<b>Prediction of Level-Crossings</b>	<b>283</b>
9.1	Partial Likelihood	284
9.2	The Logistic Model	286
9.2.1	Logistic Autoregression	287
9.2.2	Some Typical Calculations	288
9.3	Large Sample Theory	289
9.3.1	Asymptotic Relative Efficiency	293
9.4	Applications of Logistic Regression	294
9.4.1	Threshold Exceedances by Rain Rate	294
9.4.2	Application to Rainfall-Runoff Data	296
9.5	Problems and Complements	299
<b>Appendix A</b>	<b>Some Data Sets</b>	<b>302</b>
A.1	Hourly GATE Phase I Data	302
A.2	Annual Mean Air Temperature	307
A.3	Canadian Lynx Data	308
<b>Index</b>		<b>309</b>