

## Stochastic Dynamical Systems

Concepts, Numerical Methods, Data Analysis

Josef Honerkamp

TRANSLATED BY Katja Lindenberg



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