NUMERICAL METHODS USING MATLAB

Dr John Penny George Lindfield Department of Mechanical Engineering, Aston University



IS HORWOOD NEW YORK LONDON TORONTO SYDNEY TOKYO SINGAPORE

Contents

Preface

1	An i	ntroduction to MATLAB	
	1.1	The software package MATLAB	1
	1.2	MATLAB on personal computers and workstations	2
	1.3	Matrices and matrix operations in MATLAB	3
	1.4	Using the MATLAB operator \ for matrix division	4
	1.5	Manipulating the elements of a matrix	4
	1.6	Transposing matrices	5
	1.7	Special matrices	6
	1.8	Generating matrices with specified element values	7
	1.9	Some special matrix operations	7
	1.10	Element-by-element operations	7
	1.11	Input and output in MATLAB	8
	1.12	MATLAB graphics	10
	1.13	Three-dimensional graphics	14
	1.14	Scripting in MATLAB	16
	1.15	Functions in MATLAB	20
	1.16	User-defined functions	21
	1.17	Some pitfalls in MATLAB	23
	1.18	Speeding up calculations in MATLAB	24
	Prob	lems	25

xi

Con	tents
-----	-------

2	Line	ar equations and eigensystems	
	2.1	Introduction	28
	2.2	Linear equation systems	31
	2.3	MATLAB operators $\ \ d = b$	36
	2.4	Accuracy of solutions and ill-conditioning	41
	2.5	Elementary row operations	44
	2.6	Solution of $Ax = b$ by Gaussian elimination	45
	2.7	LU decomposition	47
	2.8	Cholesky decomposition	51
	2.9	QR decomposition	53
	2.10	Singular value decomposition	56
	2.11	The pseudo-inverse	60
	2.12	Over-determined systems	62
	2.13	Iterative methods	65
	2.14	Sparse matrices	66
	2.15	The eigenvalue problem	75
	2.16	The MATLAB function eig	78
	2.17	Summary	82
	Prob	lems	82
3	Root	s of equations	
	3.1	Introduction	85
	3.2	The nature of solutions to non-linear equations	87
	3.3	The bisection algorithm	89
	3.4	Iterative or fixed point methods	89
	3.5	The convergence of iterative methods	90
	3.6	Ranges for convergence and chaotic behaviour	92
	3.7	Newton's method	94
	3.8	Schroder's method	98
	3.9	Numerical problems	99
	3.10	The MATLAB function fzero and comparative studies	102
	3.11	Methods for finding all the roots of a polynomial	103
	3.12	Bairstow's method	103
	3.13	Laguerre's method	107
	3.14	Solving systems of non-linear equations	108

5.15	Laguerre s method	107
3.14	Solving systems of non-linear equations	108
3.15	Broyden's method for solving non-linear equations	112
3.16	Comparing the Newton and Broyden methods	115
3.17	Summary	116
Probl	lems	116

vi

3

Con	tents
COL	i cinto

4	Diffe	erentiation and integration	
	4.1	Introduction	120
	4.2	Numerical differentiation	121
	4.3	Numerical integration	124
	4.4	Simpson's rule	126
	4.5	Newton-Cotes formulae	129
	4.6	Romberg integration	131
	4.7	Gaussian integration	133
	4.8	Infinite ranges of integration	136
	4.9	Gauss-Chebyshev formulae	140
	4.10	Filon's sine and cosine formulae	0 141
	4.11	Problems in the evaluation of integrals	145
	4.12	Test integrals	147
	4.13	Repeated integrals	149
	4.14	Simpson's rule for repeated integrals	150
	4.15	Gaussian integration for repeated integrals	152
	4.16	Summary	154
	Probl	lems	155

5 Differential equations

5.1	Introduction	159	
5.2	Euler's method	161	
5.3	The problem of stability	163	
5.4	The trapezoidal method	166	
5.5	Runge–Kutta methods	168	
5.6	Predictor-corrector methods	173	
5.7	Hamming's method and the use of error estimates	175	
5.8	Error propagation in differential equations	177	
5.9	The stability of particular numerical methods	178	
5.10	Systems of simultaneous differential equations	180	
5.11	The Lorenz equations	184	
5.12	The predator-prey problem	186	
5.13	Differential equations applied to neural nets	187	
5.14	Higher-order differential equations	191	
5.15	Stiff equations	192	
5.16	Special techniques	195	
5.17	Extrapolation techniques	198	
5.18	Summary	200	
Probl	lems	200	

Contents

6	Bou	ndary value problems	
	6.1	Introduction	202
	6.2	The shooting method	204
	6.3	The finite difference method	206
	6.4	Two-point boundary value problems	208
	6.5	Parabolic partial differential equations	214
	6.6	Hyperbolic partial differential equations	218
	6.7	Elliptic partial differential equations	221
	6.8	Summary	227
	Prob	lems	228
_		and a sense from the activity of the second second second	
7	Fitti	ng functions to data	
	7.1	Introduction	231
	7.2	Interpolation using polynomials	231
	7.3	Interpolation using splines	234
	7.4	Fourier analysis of discrete data	238
	7.5	Fitting functions to data: least squares criteria	252
	7.6	Polynomial least squares	254
	7.7	A problem with polynomial least squares fitting	257
	7.8	General least squares	258
	7.9	Transforming data	260
	7.10	Summary	263
	Prob	lems	264
8	Opti	misation methods	
	8.1	Introduction	269
	8.2	Linear programming problems	269
	8.3	The conjugate gradient method	276
	8.4	The conjugate gradient algorithm for solving	
		linear equation systems	281
	8.5	Genetic algorithms	284
	8.6	Summary	295
	Prob	lems	295

viii

Contents

Appendix 1 – Matrix algebra

A1.1	Introduction	297
A1.2	Matrices and vectors	297
A1.3	Some special matrices	298
A1.4	Determinants	299
A1.5	Matrix operations	300
A1.6	Complex matrices	301
A1.7	Matrix properties	302
A1.8	Some matrix relationships	302
A1.9	Eigenvalues	303
A1.10	Definition of norms	303
A1.11	Reduced row echelon form	304
Appendix	2 – List of MATLAB functions	305
References	5	309
Solutions to problems		313
Index		323

ix