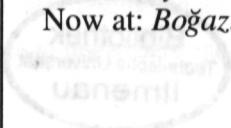


# **Change Detection and Input Design in Dynamical Systems**

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