# Nonlinear Equations in Physics and Mathematics

Proceedings of the NATO Advanced Study Institute held in Istanbul, Turkey, August 1–13, 1977

edited by

A.O. BARUT

Department of Physics, University of Colorado, Boulder, Colo., U.S.A.

> UNIVERSITÄTSBIBLIOTHEK HANNOVER

TECHNISCHE INFORMATIONSBIBLIOTHEK



## D. Reidel Publishing Company

Dordrecht : Holland / Boston : U.S.A. / London : England

Published in cooperation with NATO Scientific Affairs Division

# RN 2937 1407

#### TABLE OF CONTENTS

Preface	ix
PART I - DYNAMICAL SYSTEMS AND INVERSE SCATTERING PROBLEMS	
INTEGRABLE MANY-BODY PROBLEMS / F.Calogero	3
1. Introduction	3
2. Many-Body Problems Solvable by the Lax Trick	3
3. Motion of Poles of Nonlinear Partial Differential	
Equations and Related Many-Body Problems	27
4. Motion of Zeros of Linear Evolution Equations and	
Related Integrable Many-Body Problems	40
INVERSE SCATTERING PROBLEMS FOR NONLINEAR	
APPLICATIONS / P.C.Sabatier	55
1. Introduction	55
2. Scattering Problems before Nonlinear Applications	56
3. Structure of the I.S.P. Solution : The	
Transformation Operator	63
4. Structure of the I.S.P. Solution : The Integral	
Equation	66
5. Construction of Nonlinear Equations	76
6. A Concluding Diagram	78
SOLUTIONS OF NONLINEAR EQUATIONS SIMULATING PAIR	
PRODUCTION AND PAIR ANNIHILATION / A.O.Barut	81
THE TWO-TIME METHOD APPLIED TO SLOWLY EVOLVING	
OSCILLATING SYSTEMS / J.Robert Buchler	85
1. Introduction	85
2. The Two-Time Method	88
3. The Harmonic Approximation and Vibrational	
Stability Analysis	. 91
4. Summary	95

PART II - SOLITONS

SOLITONS IN PHYSICS / R.K.Bullough	99
1. Introductory Mathematics	99
2. Applications of Solitons to Nonlinear Physics	114
3. Some Particular Applications of Solitons in Physics	117
4. Quantized Solitons	133
SOLITONS AND GEOMETRY / Fernando Lund	143
1. Introduction	143
2. Geometry	145
3. Physics	153
4. Solitons	163
HIROTA'S METHOD OF SOLVING SOLITON-TYPE	
EQUATIONS / P.J.Caudrey	177
1. The Korteweg-de Vries Equation	177
2. The Sine-Gordon Equation	181
3. The Double-Sine-Gordon Equation	183
4. A Hierarchy of KdV Equations	185
5. Polynomial Conserved Densities	190
BOAT AN ALMTAN AND MARKINE MEANINE THE THE AND	
PROLONGATION STRUCTURE TECHNIQUES FOR THE NEW HIERARCHY	
OF KORTEWEG-DE VRIES EQUATIONS / R.K.Dodd	193
PERTURBATION THEORY FOR THE DOUBLE SINE-GORDON	
EQUATION / A.L.Mason	205
PART III - DISCRETE SYSTEMS AND CONTINUUM MECHANICS	
DATHI BUE TRANSCONDENTS AND CONTINUE DUNGETONS OF THE	
FAINLEVE TRANSCENDENTS AND SCALING FUNCTIONS OF THE	
TWO-DIMENSIONAL ISING MODEL / Craig A.Tracy	221
J. Introduction	221
2. Iwo-Dimensional Ising Model	222
3. Scaling Limit and Scaling Functions	223
4. Explicit Formulas for $F_{\pm}(x)$	224
5. Painlevé Transcendents	228
STATISTICAL MECHANICS OF NONLINEAR LATTICE DYNAMIC	
MODELS EXHIBITING PHASE TRANSITIONS / T.Schneider	239
1. Introduction	239
2. The Models and their Continuum Limit, the Equations	
of Motion and Particular Solutions	241
3. Dynamic Variables, Conservation Laws and Spectral	
Densities	247
4. Molecular-Dynamics Technique	247
5. Molecular-Dynamics Results: Static and Dynamic	243
Equilibrium Properties	251
6. Nonlinear Heat-Pulse Propagation	260
	£.00

vi

#### TABLE OF CONTENTS

NONLOCAL CONTINUUM MECHANICS AND SOME		
APPLICATIONS / A.Cemal Eringen		271
1. Introduction		271
2. Balance Laws		27.4
3. Constitutive Equations		276
4. Thermodynamic Restrictions		279
5. Linear Theory	and the second	281
6. Determination of Nonlocal Elastic Moduli		285
7. Surface Waves	· · · · · ·	289
8. Screw Dislocation		291
9. Fracture Mechanics		295
10. Nonlocal Fluid Mechanics and Turbulence	÷ 1	303

### PART IV - NONLINEAR FIELD THEORIES AND QUANTIZATION

1. The Classical Theory       322         2. Symmetry Transformations       324         3. Quantum Mechanics of the Free Field Equation       324         4. Compactification of Time       325         5. Perturbation Expansion       326         6. The Classical Scattering Theory       327         7. The Quantization       328         8. The Commutation Relations       332         9. Quantization in Analogy to the Thirring Model       332         9. Quantization in Analogy to the Thirring Model       332         CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD       EQUATIONS / A.O.Barut         S. Nonlinear Fields       337         3. Multicomponent Fields       341         4. Nonlinear Chiral Fields       341         5. Physical Interpretation and Applications       343         6. Choice of Nonlinear Model       344         NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN       APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and         R.Jengo       347         1. A General Field Theoretical Problem       347         2. An Application of the Canonical Formalism       349         NONLINEAR FIELD EQUATIONS AND COLLECTIVE       PHENOMENA / H.Kleinert         9. Presence of Pairing Force       366         3. Fresence of Pairing	QUANTIZATION OF A NONLINEAR FIELD EQUATION / L.Castell	321
2. Symmetry Transformations3243. Quantum Mechanics of the Free Field Equation3244. Compactification of Time3255. Perturbation Expansion3266. The Classical Scattering Theory3277. The Quantization3288. The Commutation Relations3329. Quantization in Analogy to the Thirring Model332CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD200EQUATIONS / A.O.Barut3351. Linear Fields3373. Multicomponent Fields3414. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE355PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion355	1. The Classical Theory	322
3. Quantum Mechanics of the Free Field Equation       324         4. Compactification of Time       325         5. Perturbation Expansion       326         6. The Classical Scattering Theory       327         7. The Quantization       328         8. The Commutation Relations       332         9. Quantization in Analogy to the Thirring Model       332         CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD       335         1. Linear Fields       335         2. Nonlinear Fields       337         3. Multicomponent Fields       341         4. Nonlinear Chiral Fields       341         5. Physical Interpretation and Applications       343         6. Choice of Nonlinear Model       344         NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN       347         7. A General Field Theoretical Problem       347         7. An Application of the Canonical Formalism       349         NONLINEAR FIELD EQUATIONS AND COLLECTIVE       349         NONLINEAR FIELD EQUATIONS AND COLLECTIVE       355         1. Introduction       355         2. A simple Model       356         3. Presence of Pairing Force       366         4. Conclusion       355	2. Symmetry Transformations	324
4. Compactification of Time       325         5. Perturbation Expansion       326         6. The Classical Scattering Theory       327         7. The Quantization       328         8. The Commutation Relations       332         9. Quantization in Analogy to the Thirring Model       332         CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD       335         1. Linear Fields       335         2. Nonlinear Fields       335         3. Multicomponent Fields       341         4. Nonlinear Chiral Fields       341         5. Physical Interpretation and Applications       343         6. Choice of Nonlinear Model       344         NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN       347         1. A General Field Theoretical Problem       347         2. An Application of the Canonical Formalism       349         NONLINEAR FIELD EQUATIONS AND COLLECTIVE       355         1. Introduction       355         2. A Simple Model       355         3. Fresence of Pairing Force       366         4. Conclusion       356	3. Quantum Mechanics of the Free Field Equation	324
5. Perturbation Expansion 326 6. The Classical Scattering Theory 327 7. The Quantization 328 8. The Commutation Relations 332 9. Quantization in Analogy to the Thirring Model 332 CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD EQUATIONS / A.O.Barut 335 1. Linear Fields 337 3. Multicomponent Fields 341 4. Nonlinear Chiral Fields 341 5. Physical Interpretation and Applications 343 6. Choice of Nonlinear Model 344 NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 355 3. A Simple Model 356 3. Presence of Pairing Force 366 4. Conclusion 373	4. Compactification of Time	325
6. The Classical Scattering Theory       327         7. The Quantization       328         8. The Commutation Relations       332         9. Quantization in Analogy to the Thirring Model       332         CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD       335         EQUATIONS / A.O.Barut       335         1. Linear Fields       337         3. Multicomponent Fields       337         3. Multicomponent Fields       341         4. Nonlinear Chiral Fields       341         5. Physical Interpretation and Applications       343         6. Choice of Nonlinear Model       344         NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN       347         1. A General Field Theoretical Problem       347         2. An Application of the Canonical Formalism       349         NONLINEAR FIELD EQUATIONS AND COLLECTIVE       347         2. A Simple Model       355         3. Introduction       355         3. Fresence of Pairing Force       366         4. Gonclusion       356	5. Perturbation Expansion	326
7. The Quantization3288. The Commutation Relations3329. Quantization in Analogy to the Thirring Model332CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD335EQUATIONS / A.O.Barut3351. Linear Fields3352. Nonlinear Fields3373. Multicomponent Fields3414. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE3551. Introduction3552. A Simple Model3563. Fresence of Pairing Force3664. Conclusion3563. Fresence of Pairing Force366	6. The Classical Scattering Theory	327
8. The Commutation Relations 9. Quantization in Analogy to the Thirring Model 332 CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD EQUATIONS / A.O.Barut 1. Linear Fields 335 1. Linear Fields 337 3. Multicomponent Fields 341 4. Nonlinear Chiral Fields 343 6. Choice of Nonlinear Model NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 347 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 357 358 359 350 350 350 350 355 355 355 355	7. The Quantization	328
9. Quantization in Analogy to the Thirring Model 332 CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD EQUATIONS / A.O.Barut 335 1. Linear Fields 337 2. Nonlinear Fields 337 3. Multicomponent Fields 341 4. Nonlinear Chiral Fields 341 5. Physical Interpretation and Applications 343 6. Choice of Nonlinear Model 344 NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 355 2. A Simple Model 356 3. Presence of Pairing Force 366 4. Conclusion 373	8. The Commutation Relations	332
CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD EQUATIONS / A.O.Barut 335 1. Linear Fields 337 3. Nonlinear Fields 337 3. Multicomponent Fields 341 4. Nonlinear Chiral Fields 341 5. Physical Interpretation and Applications 343 6. Choice of Nonlinear Model 344 NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 355 2. A Simple Model 356 3. Presence of Pairing Force 366 4. Conclusion 373	9. Quantization in Analogy to the Thirring Model	332
EQUATIONS / A.O.Barut3351. Linear Fields3352. Nonlinear Fields3373. Multicomponent Fields3414. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	CHARACTERISTIC "QUANTA" OF NONLINEAR FIELD	
1. Linear Fields3352. Nonlinear Fields3373. Multicomponent Fields3414. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	EQUATIONS / A.O.Barut	335
2. Nonlinear Fields 337 3. Multicomponent Fields 341 4. Nonlinear Chiral Fields 341 5. Physical Interpretation and Applications 343 6. Choice of Nonlinear Model 344 NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 355 2. A Simple Model 356 3. Presence of Pairing Force 366 4. Conclusion 373	1. Linear Fields	335
3. Multicomponent Fields3414. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	2. Nonlinear Fields	337
4. Nonlinear Chiral Fields3415. Physical Interpretation and Applications3436. Choice of Nonlinear Model344NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	3. Multicomponent Fields	341
5. Physical Interpretation and Applications 6. Choice of Nonlinear Model NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 2. An Application of the Canonical Formalism NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 1. Introduction 2. A Simple Model 3. Presence of Pairing Force 4. Conclusion 343 344 344 344 344 345 347 347 348 349 347 347 347 347 347 347 347 347	4. Nonlinear Chiral Fields	341
<ul> <li>6. Choice of Nonlinear Model</li> <li>344</li> <li>NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN</li> <li>APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and</li> <li>R.Jengo</li> <li>347</li> <li>1. A General Field Theoretical Problem</li> <li>347</li> <li>2. An Application of the Canonical Formalism</li> <li>349</li> <li>NONLINEAR FIELD EQUATIONS AND COLLECTIVE</li> <li>PHENOMENA / H.Kleinert</li> <li>355</li> <li>1. Introduction</li> <li>356</li> <li>3. Presence of Pairing Force</li> <li>366</li> <li>4. Conclusion</li> </ul>	5. Physical Interpretation and Applications	343
NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	6. Choice of Nonlinear Model	344
APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and R.Jengo 347 1. A General Field Theoretical Problem 347 2. An Application of the Canonical Formalism 349 NONLINEAR FIELD EQUATIONS AND COLLECTIVE PHENOMENA / H.Kleinert 355 1. Introduction 355 2. A Simple Model 356 3. Presence of Pairing Force 366 4. Conclusion 373	NONLINEAR SCHRÖDINGER EQUATION WITH SOURCES : AN	
R.Jengo3471. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE9PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	APPLICATION OF THE CANONICAL FORMALISM / L.Girardello and	
1. A General Field Theoretical Problem3472. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE9PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	R.Jengo	347
2. An Application of the Canonical Formalism349NONLINEAR FIELD EQUATIONS AND COLLECTIVE355PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	1. A General Field Theoretical Problem	347
NONLINEAR FIELD EQUATIONS AND COLLECTIVEPHENOMENA / H.Kleinert3551. Introduction2. A Simple Model3. Presence of Pairing Force3664. Conclusion373	2. An Application of the Canonical Formalism	349
PHENOMENA / H.Kleinert3551. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	NONLINEAR FIELD EQUATIONS AND COLLECTIVE	
1. Introduction3552. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	PHENOMENA / H.Kleinert	355
2. A Simple Model3563. Presence of Pairing Force3664. Conclusion373	1. Introduction	355
3. Presence of Pairing Force3664. Conclusion373	2. A Simple Model	356
4. Conclusion 373	3. Presence of Pairing Force	366
	4. Conclusion	373

vii

NONPERTURBATIVE SELF-INTERACTIONS, SOLITARY WAVES	
AND OTHERS / Philip B.Burt	375
1. Introduction	375
2. Nonperturbative, Self-Interacting Quantum Fields	376
3. Solitary Wave Propagators	381
4. Solitary Waves and Others	387
5. Concluding Remarks	396
BOUND STATES OF FERMIONS IN EXTERNAL AND	200
1 Solutions of the Direct Fruction	399
2. Quantum Field Theory of Spin-2 Particles in	400
Strong External Fields	414
3. Supercharged Vacuum and Klein's Paradox	424
4. Strong Fields in Quantum Field Theory	434
Subject Index	469

vili