

ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

*Noncommutative Rational Series
with Applications*

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