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Differential Geometry and Mathematical Physics

Part I. Manifolds, Lie Groups and
Hamiltonian Systems

 Springer

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