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Theory of Reflection

Reflection and Transmission of Electromagnetic, Particle and Acoustic Waves

Second Edition

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