

Etienne Forest

From Tracking Code to Analysis

Generalised Courant-Snyder Theory for Any
Accelerator Model

Contents

1	Introduction	1
1.1	Dichotomous Approach Derived from Complexity	1
1.2	The Modern Way to Implement the Dichotomous Approach	2
1.3	The Induced Hierarchy Inherited from the Tracking Code	3
1.4	Give His Dues to Caesar: Dragt, Talman, and Berz	5
1.5	The Necessary Properties of the “Exact Code”.	6
1.6	Integrable Systems Are Sitting on Tori	8
1.7	The Example Code PTC: A Minimal Tutorial	10
1.7.1	The Propagator	11
1.7.2	Producing the One-Turn Taylor Map	12
1.7.3	Propagator, Propagata and Propaganda	15
	References	17
2	The Linear Transverse Normal Form: One Degree of Freedom	19
2.1	Conversion Table Between Linear and Nonlinear	19
2.2	Why Phasors and Normal Forms?	19
2.2.1	The Average of an Arbitrary Function: Need for Phasors	20
2.2.2	Linear Lattice Functions from de Moivre’s Formula	23
2.2.3	Lattice Functions as Coefficients of the Invariant	24
2.2.4	Lattice Functions as Coefficients of the Moments $\langle z_i z_j \rangle$	25
2.3	The Program <code>one_turn_orbital_map_normal_form_2d</code>	26
2.3.1	Construction of the Matrix A	26
2.3.2	The Computation of the Map.	30
2.3.3	Numerical Computation of the Canonical Transformation.	32

2.4	The Phase Advance and the Invariant	33
2.4.1	Phase Advance in a Code: Finite Map Theory	33
2.4.2	Numerical Computation of the Invariant and the Phase Advance	35
2.4.3	Computation of the Phase: Some Theory	40
2.4.4	Tracking of the Invariant: Some Hamiltonian Theory	41
	References	43
3	The Nonlinear Transverse Normal Form: One Degree of Freedom	45
3.1	The Pendulum with Exact Methods	45
3.1.1	Preliminary	45
3.1.2	The Hamiltonian of the Pendulum	46
3.1.3	The Exact Period of the Pendulum	47
3.1.4	The Frequency and the Hamiltonian as a Function of J	49
3.1.5	The Action J as a Function of θ and p	50
3.2	The Pendulum with the Numerical Methods of Cosy-Infinity	51
3.2.1	Phase Space Maps and Lie Maps	51
3.2.2	Vector Field of the Pendulum	55
3.2.3	Hamiltonian Normal Form of the Pendulum	57
3.2.4	Illustration of Normal Form Theory with FPP Tools	61
3.3	Normal Form of the Standard Map: The Algorithm for Maps	64
3.3.1	Linear Part	65
3.3.2	The Nonlinear Algorithm: Theory	66
3.4	Standard Map: Normalisation in Software	70
3.4.1	Creation of Standard Map and Using a “Canned” Normal Form	70
3.4.2	Normalisation of the Map with Vector Fields	73
3.4.3	Normalisation of the Map with Poisson Brackets	76
3.4.4	Using a Pseudo-Hamiltonian: Normalising the Logarithm	77
	References	80
4	Classification of Linear Normal Forms	81
4.1	The Full Harmonically Sinking Phase Space	81
4.1.1	Radiation: The Multidimensional Drain	82
4.1.2	De Moivre Representation of the One-Turn Matrix	83
4.1.3	Ripken Lattice Functions Representation: Invariants and Moments	84

4.1.4	De Moivre-Ripken H^a Matrices in the Symplectic Case: The Dispersions η and ζ	88
4.1.5	Kinematic Invariants of Linacs Coming Out of H^i and B^i	92
4.2	No Cavities: Jordan Normal Form	95
4.2.1	The Reasons for a No-Cavity Normal Form.	95
4.2.2	A Glance at the Nonlinear Jordan Normal Form.	96
4.2.3	The Linear Part with No Cavity	96
4.2.4	The Slip Factor and the Longitudinal Tune	99
4.3	Normal Form for the “AC” Fluctuation of a Magnet Property	101
4.3.1	Adding Pseudo Clocks to Phase Space	102
4.3.2	A Simple Calculation with FPP	103
4.3.3	The Way It Can Be Done in a Code: PTC	105
4.3.4	A Linear Complication Due to the Case of Sect. 4.2	107
4.4	The Stochastic Normal Form: Radiation Theory.	110
4.4.1	The Stochastic Map of Moments	111
4.4.2	The Normal Form of the Quadratic Moment Map.	112
4.4.3	Numerical Example	114
	References	119
5	Nonlinear Normal Forms	121
5.1	What Do I Mean by Nonlinear?	121
5.2	The (Damped) Pseudo-Harmonic Oscillator Case	123
5.3	Derivation of Eq. (4.74): Magnet Modulation Revisited	126
5.4	One Resonance Orbital Normal Form	129
5.4.1	The Naive Dragt Approach: Raising the Map to a Power	130
5.4.2	General Approach: Leaving One Resonance	132
5.4.3	The Co-moving Map	135
5.4.4	The Instructive Resonant Case $\mathcal{M} = \mathcal{R}_{\nu=\frac{1}{4}+\delta} \exp\left(\frac{k_i}{3}x^3 + \frac{k_o}{4}x^4\right)$	137
5.5	A Map with a Limit Cycle: Akin to a Resonance.	144
5.5.1	The Computation of Limit Cycles	144
5.5.2	Computation of \mathcal{R} of Eq. (5.62): Co-sinking Map	147
5.5.3	Example Program for the Limit Cycle of Fig. 5.5b.	147
	References	149
6	Spin Normal Form	151
6.1	Introductory Verbiage on Spin in the Code PTC	151
6.2	The Normal Form for Spin on the Closed Orbit: n_0	153

6.3	The Nonlinear Normal Form for the Invariant Spin Field $n(z)$	155
6.3.1	Short History and Comments on the Mysterious ISF $n(z)$	155
6.3.2	The Normal Form and Why We Get $n(z)$ from It	156
6.3.3	The Algorithm for the Spin Normal Form	158
6.3.4	A Code Implementation for the Spin Normal Form	162
6.4	Leaving One Resonance in the Spin Map	164
6.4.1	Two Cases for the Spectator Spin: Orbital and Spin-Orbit	164
6.5	The Abell-Barber Co-moving Map for the Spin Orbit Case	166
6.5.1	N_0 and the Co-moving Spin Map	166
6.5.2	Solution for the ISF n of the One-Resonance Map of Eq. (6.57)	168
6.5.3	Numerical Behaviour of the One-Resonance ISF	171
	References	175
7	The Nonlinear Spin-Orbital Phase Advance: The Mother of All Algorithms	177
7.1	Introductory Verbiage on Phase Advance and “Canonisation”.	177
7.2	A Little Notation Hurdle Due to the Jordan Normal Form	179
7.3	My Choice for the Fixed Point Map	180
7.3.1	The Trivial Case	181
7.3.2	The Case of a Jordan Normal Form	181
7.4	The Linear Transformation $A(s)$	182
7.5	My Choice for the Nonlinear Transformation b	185
7.6	My Choice for the Spin Transformation D	185
7.7	The Canonisation in the Example Code of Appendix M	186
7.7.1	Setting Up the Example	186
7.7.2	Finding the One-Turn Map and Normalising It	187
7.7.3	Example of Canonisations in the “Courant-Snyder Loop”.	188
7.8	The Phase Advance, Its Freedom and the Code of Appendix M	191
7.8.1	The Case of Hamiltonian Perturbation Theory	191
7.8.2	The Case of Map Perturbation Theory: Orbital and Spin	194
7.8.3	Description of the Phase Advance Loop of Appendix M	195
7.8.4	Example of Something Being Computed in the Courant-Snyder Loop.	197
	References	203

8	Deprit-Guignard Perturbation Theory Faithful to the Code	205
8.1	How About Hamiltonian Perturbation Theory?	205
8.2	Defining a Time-Like Variable	206
8.3	Using a Courant-Snyder Type Phase Advance	207
8.4	Using a Constant Phase Advance	210
8.5	Code Example for Sect. 8.3: Using the Courant-Snyder Phase Advance	212
8.6	Code Example for Sect. 8.4: Using a Constant Phase Advance	218
8.7	Normalising the $\theta(s)$ -Dependent Equations of Motion: Deprit-Guignard Approach	225
8.7.1	Transforming the Equations of Motions.	226
8.7.2	The Actual Deprit-Guignard Normal Form	228
8.7.3	Example Code for a Deprit-Guignard Normalisation	230
8.7.4	Numerical Example $\langle x \rangle$: Analytical, Guignard and Map	234
8.7.5	The Final Strategy: Be Prepared to Mix Everything!	234
	References	235
9	Here Is the Conclusion of This Book.	237
9.1	Conclusion	237
9.2	Exclusion	241
9.2.1	A Deeper Discussion About Guignard Normalisation	241
9.2.2	Synchro-Betatron Effects	242
10	Phasors Basis: Why Do I Reject Symplectic Phasors?	245
11	The Logarithm of a Map	247
	Reference	250
12	Stroboscopic Average for the ISF Vector \mathbf{n}	251
	References	253
13	Hierarchy of Analytical Methods	255
13.1	Green's Function Method	256
13.1.1	The Rules of Analytical Perturbation Theory with Maps	256
13.1.2	The Actual Calculation with Maps	256
13.2	Fourier Mode Calculations with the Hamiltonian	260
13.2.1	Changing the Time-Like Variable into a Phase Advance	260
13.2.2	The Fourier Method Approach: Guignard	261
13.3	Numerical Example $\langle x \rangle$: Analytical, Guignard and Map	264
	References	268

Appendix A: The Hardwired ALS Lattice	269
Appendix B: Program for one_turn_orbital_map	273
Appendix C: Program one_turn_orbital_map_normal_form_2d	275
Appendix D: Program one_turn_orbital_map_phase_ad	279
Appendix E: Program Pendulum	283
Appendix F: Program standard_map	287
Appendix G: Program one_turn_cavity_map	293
Appendix H: Program radiation_map	299
Appendix I: Program modulated_map	303
Appendix J: Program modulated_map_Jordan.	307
Appendix K: Program one_resonance_map	311
Appendix L: Program very_damped_map	317
Appendix M: Program spin_phase_advance_isf	319
Appendix N: Program hamitonian_guignard_cs.f90	325
Appendix O: Program hamitonian_guignard.f90	331
Appendix P: Program hamiltonian_guignard_1df.f90	337
Appendix Q: Program hamiltonian_guignard_1df_x.f90	341
Index of Links to Useful Concepts and Formulae	347