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Non-commuting Variations in Mathematics and Physics

A Survey

 Springer

NOTES ON THE NONCOMMUTING VARIATIONS.

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- (2) Chapter 2. Lagrangian Field Theory with the non-commuting (NC) variations - p.17,
- (3) Chapter 3. Vertical connections in the Configurational bundle and the NC-variations - p.53,
- (4) Chapter 4. K -twisted prolongations and μ -symmetries (by works of Muriel, Romero, Gaeta, Morando, etc.) - p.79 ,
- (5) Chapter 5. Applications: Holonomic and non-Holonomic Mechanics, H. Kleinert Action principle, Uniform Materials, and the Dissipative potentials - p.111,
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