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Advanced Geometrical Optics



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Part I A New Light on Old Geometrical Optics (Raytracing Equations of Geometrical Optics)

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