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# Achieving Consensus in Robot Swarms

Design and Analysis of Strategies  
for the best-of- $n$  Problem

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