

Insights into Game Theory

An Alternative Mathematical
Experience

EIN-YA GURA
AND
MICHAEL MASCHLER

 **CAMBRIDGE**
UNIVERSITY PRESS

Contents

Preface	<i>page xi</i>
Introduction	xii
1 Mathematical Matching	1
1.1 Introduction	1
1.2 The Matching Problem	2
1.3 Exercises	7
1.4 Further Examples	10
1.5 Exercises	13
1.6 A Procedure for Finding Stable Matching Systems (The Gale–Shapley Algorithm)	15
1.7 Exercises	18
1.8 A Stable Matching System Always Exists	19
1.9 The Maximum Number of Courtship Stages in the Gale–Shapley Algorithm	21
1.10 Generalization	26
1.11 Exercises	33
1.12 The Gale–Shapley Algorithm and the Assignment Problem	37
1.13 Exercises	40
1.14 Optimality	43
1.15 Exercises	49
1.16 Condition for the Existence of a Unique Stable Matching System	52
1.17 Exercises	54
1.18 Discussion	55
1.19 Review Exercises	56

2	Social Justice	59
2.1	Presentation of the Problem	59
2.2	Mathematical Description of the Problem	62
2.3	Exercises	64
2.4	Social Choice Function	67
2.5	Axioms for the Social Choice Function	77
2.6	Exercises	80
2.7	What Follows from Axioms 1–4?	81
2.8	Exercises	85
2.9	Arrow's Theorem	87
2.10	What Next?	92
2.11	Review Exercises	93
3	The Shapley Value in Cooperative Games	97
3.1	Introduction	97
3.2	Cooperative Games	98
3.3	Important Examples of Coalition Function Games	101
3.4	Exercises	105
3.5	Additive Games	106
3.6	Superadditive Games	107
3.7	Majority Games	108
3.8	Exercises	112
3.9	Symmetric Players	113
3.10	Exercises	115
3.11	Null Players	116
3.12	Exercises	117
3.13	The Sum of Games	118
3.14	Exercises	121
3.15	The Shapley Value	124
3.16	Exercises	133
3.17	Dissolving a Partnership	133
3.18	Exercises	141

3.19	The Shapley Value as the Average of Players' Marginal Contributions	142
3.20	Exercises	146
3.21	The Shapley Value as a Player's Index of Power in Weighted Majority Games	148
3.22	Exercises	153
3.23	The Shapley–Shubik Index as an Index for the Analysis of Parliamentary Phenomena	153
3.24	Exercises	156
3.25	The Security Council	156
3.26	Exercises	158
3.27	Cost Games	159
3.28	Exercises	162
3.29	Review Exercises	164
4	Analysis of a Bankruptcy Problem from the Talmud	166
4.1	Introduction	166
4.2	The Contested Garment	168
4.3	Exercises	171
4.4	A Physical Interpretation of the Contested-Garment Principle	172
4.5	Exercises	176
4.6	A Bankruptcy Problem from the Talmud	177
4.7	Exercises	180
4.8	Existence and Uniqueness	182
4.9	Divisions Consistent with the Contested-Garment Principle	186
4.10	Exercises	191
4.11	Consistency	192
4.12	Exercises	194
4.13	Rif's Law of Division	194
4.14	Exercises	196
4.15	Proportional Division	196

4.16	O'Neill's Law of Division	197
4.17	Exercises	200
4.18	Discussion	201
4.19	Review Exercises	203
A	Answers to the Exercises	205
A.1	Chapter 1	205
A.2	Chapter 2	213
A.3	Chapter 3	220
A.4	Chapter 4	229
	Bibliography	233
	Index	235