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# Vibrations of Rotating Machinery

Volume 1. Basic Rotordynamics:  
Introduction to Practical Vibration Analysis

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