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# Identifying Stock Market Bubbles

Modeling Illiquidity Premium and Bid-Ask Prices of Financial Securities

# Contents

|          |                                                                |           |
|----------|----------------------------------------------------------------|-----------|
| <b>1</b> | <b>Introduction .....</b>                                      | <b>1</b>  |
|          | Reference .....                                                | 3         |
| <b>2</b> | <b>Review on Research Conducted .....</b>                      | <b>5</b>  |
| 2.1      | Inventory Models .....                                         | 5         |
| 2.2      | Information Models .....                                       | 7         |
| 2.2.1    | Informed Traders vs. Market Makers .....                       | 7         |
| 2.2.2    | Bid-Ask Spread as the Statistical Model .....                  | 8         |
| 2.2.3    | Introduction of Transaction Costs .....                        | 10        |
| 2.3      | Conic Finance .....                                            | 10        |
|          | References .....                                               | 11        |
| <b>3</b> | <b>Theory of Conic Finance .....</b>                           | <b>13</b> |
| 3.1      | Conic Finance .....                                            | 13        |
| 3.2      | Conic Finance in Practice .....                                | 14        |
| 3.3      | Distortion Functions .....                                     | 17        |
| 3.3.1    | Minvar .....                                                   | 18        |
| 3.3.2    | Maxvar .....                                                   | 18        |
| 3.3.3    | Maxminvar .....                                                | 19        |
| 3.3.4    | Minmaxvar .....                                                | 20        |
| 3.3.5    | Wang Transform .....                                           | 20        |
|          | References .....                                               | 21        |
| <b>4</b> | <b>Stock Prices Follow a Brownian Motion .....</b>             | <b>23</b> |
| 4.1      | Geometric Brownian Motion: Introduction .....                  | 23        |
| 4.2      | Option Pricing with Geometric Brownian Motion .....            | 25        |
| 4.3      | Bid-Ask Prices of European Options Under Brownian Motion ..... | 28        |
| 4.4      | Data and Numerical Application .....                           | 30        |
|          | References .....                                               | 34        |

|          |                                                                          |     |
|----------|--------------------------------------------------------------------------|-----|
| <b>5</b> | <b>Stock Prices Follow a Double Exponential Jump-Diffusion Model</b>     | 37  |
| 5.1      | Details of Jump-Diffusion Models                                         | 37  |
| 5.1.1    | Reasons for Using Jump-Diffusion Models                                  | 38  |
| 5.1.2    | Leptokurticity of Returns                                                | 38  |
| 5.1.3    | Exponential and Power-Type Tails                                         | 39  |
| 5.1.4    | Implied Volatility Smile                                                 | 40  |
| 5.1.5    | Alternatives for Black-Scholes Model                                     | 40  |
| 5.1.6    | Unique Characteristics of Jump Diffusion                                 | 41  |
| 5.2      | Jump-Diffusion Model                                                     | 42  |
| 5.3      | Distribution Function of Jump Process                                    | 43  |
| 5.4      | Distribution Function of $L_t$                                           | 56  |
| 5.5      | Risk-Neutral Dynamics                                                    | 65  |
|          | References                                                               | 70  |
| <b>6</b> | <b>Numerical Implementation and Parameter Estimation Under KOU Model</b> | 73  |
| 6.1      | Estimation Method: Theoretical Background                                | 73  |
| 6.1.1    | Maximum-Likelihood Estimation                                            | 73  |
| 6.1.2    | Generalized Method of Moments                                            | 74  |
| 6.1.3    | Characteristic Function Estimation Method                                | 76  |
| 6.1.4    | Monte-Carlo Simulation                                                   | 80  |
| 6.2      | Estimation Methods: Numerical Application                                | 81  |
| 6.2.1    | Characteristic Function and Moments of Kou Model                         | 81  |
| 6.2.2    | Simulation of Kou Model                                                  | 82  |
| 6.2.3    | Cumulant Matching Method                                                 | 83  |
| 6.2.4    | Maximum-Likelihood Estimation                                            | 85  |
| 6.2.5    | Method of Characteristic Function Estimation                             | 86  |
| 6.3      | Bid-Ask Prices of European Options Under Kou Model                       | 90  |
| 6.4      | Data and Numerical Application of the Estimation Results of Kou Model    | 92  |
|          | References                                                               | 96  |
| <b>7</b> | <b>Illiquidity Premium and Connection with Financial Bubbles</b>         | 97  |
| 7.1      | A Brief History of Financial Bubbles                                     | 97  |
| 7.1.1    | Tulip Mania                                                              | 97  |
| 7.1.2    | South-Sea Bubble                                                         | 98  |
| 7.1.3    | 1929 Great Depression                                                    | 98  |
| 7.1.4    | The Tech Bubble                                                          | 101 |
| 7.1.5    | Subprime Mortgage Bubble                                                 | 102 |
| 7.2      | Illiquidity Premium vs. Financial Bubbles                                | 104 |
| 7.3      | Comparison with Other Bubble-Detection Techniques                        | 109 |
| 7.3.1    | Value in Economics                                                       | 109 |
| 7.3.2    | Rational Bubbles                                                         | 110 |
| 7.3.3    | Heterogeneous Beliefs Bubbles                                            | 112 |
| 7.3.4    | Behavioral Bubbles                                                       | 112 |

- 7.4 Log-Periodic Power Law Model vs. Illiquidity Premium ..... 114
- 7.5 Investment Management and Illiquidity Premium ..... 116
- References ..... 116
- 8 Conclusion and Future Outlook..... 119**
- Appendix A Some Distributions Under Wang Transform ..... 123**
- Appendix B Deriving Bid and Ask Prices for Options Under**  
**Brownian Motion Assumptions ..... 125**
  - B.1 Prices of a European Call Options ..... 125
    - B.1.1 Bid Price ..... 125
    - B.1.2 Ask Price ..... 128
  - B.2 Prices of a European Put Options ..... 129
    - B.2.1 Bid Price ..... 130
    - B.2.2 Ask Price ..... 130