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# Algorithmics for Hard Problems

Introduction  
to Combinatorial Optimization,  
Randomization, Approximation,  
and Heuristics

Second Edition

With 71 Figures



Springer

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