Modern Trends in الله Financial Engineering

Stochastic Drawdowns

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NEW JERSEY · LONDON · SINGAPORE · BEIJING · SHANGHAI · HONG KONG · TAIPEI · CHENNAI · TOKYO

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