Contents

Lis	t of]	Figures xi	X
1	Intr	roduction and Preview	1
	1.1	Preview of the book / 5	
2	Ent	ropy, Relative Entropy and Mutual Information	2
	2.1	Entropy / 12	
	2.2	Joint entropy and conditional entropy / 15	
	2.3	Relative entropy and mutual information / 18	
	2.4	Relationship between entropy and mutual information / 19	
	2.5	Chain rules for entropy, relative entropy and mutual information / 21	
	2.6	Jensen's inequality and its consequences / 23	
	2.7	The log sum inequality and its applications / 29	
	2.8	Data processing inequality / 32	
	2.9	The second law of thermodynamics / 33	
	2.10	Sufficient statistics / 36	
	2.11	Fano's inequality / 38	
		Summary of Chapter 2 / 40	
		Problems for Chapter 2 / 42	
		Historical notes / 49	
3	The	Asymptotic Equipartition Property 5	0
	3.1	The AEP / 51	

xiv CONTENTS

3.2 3.3	Consequences of the AEP: data compression / 53 High probability sets and the typical set / 55 Summary of Chapter 3 / 56 Problems for Chapter 3 / 57 Historical notes / 59	
Ent	ropy Rates of a Stochastic Process	60
4.1	Markov chains / 60	
4.2	Entropy rate / 63	
4.3	Example: Entropy rate of a random walk on a weighted graph / 66	
4.4	Hidden Markov models / 69	
	Summary of Chapter 4 / 71	
	Problems for Chapter 4 / 72	
	Historical notes / 77	
Dat	a Compression	78
5.1	Examples of codes / 79	
5.2	Kraft inequality / 82	
5.3	Optimal codes / 84	
5.4	Bounds on the optimal codelength / 87	
5.5	Kraft inequality for uniquely decodable codes / 90	
5.6	Huffman codes / 92	
5.7	Some comments on Huffman codes / 94	
5.8	Optimality of Huffman codes / 97	
5.9	Shannon-Fano-Elias coding / 101	
5.10	5	
5.11	The state of the s	
5.12	coins / 110	
	Summary of Chapter 5 / 117	
	Problems for Chapter 5 / 118	
	Historical notes / 124	
Gan	nbling and Data Compression	125
6.1	The horse race / 125	
6.2	Gambling and side information / 130	
6.3	Dependent horse races and entropy rate / 131	
6.4	The entropy of English / 133	
6.5	Data compression and gambling / 136	

CONTENTS xv

6.6 Gambling estimate of the entropy of English / 138

7

8

	Summary of Chapter 6 / 140	
	Problems for Chapter 6 / 141	
	Historical notes / 143	
Koln	nogorov Complexity	144
7.1	Models of computation / 146	
7.2	Kolmogorov complexity: definitions and examples / 147	
7.3	Kolmogorov complexity and entropy / 153	
7.4	Kolmogorov complexity of integers / 155	
7.5	Algorithmically random and incompressible sequences / 156	
7.6	Universal probability / 160	
7.7	The halting problem and the non-computability of Kolmogorov complexity / 162	
7.8	Ω / 164	
7.9	Universal gambling / 166	
7.10		
7.11		
7.12	8	
	Summary of Chapter 7 / 178	
	Problems for Chapter 7 / 180	
	Historical notes / 182	
Char	anal Canacity	183
	nnel Capacity	100
8.1	Examples of channel capacity / 184	
8.2	Symmetric channels / 189	
8.3	Properties of channel capacity / 190	
8.4	Preview of the channel coding theorem / 191	
8.5	Definitions / 192	
8.6	Jointly typical sequences / 194	
8.7	The channel coding theorem / 198 Zero-error codes / 203	
8.8		
8.9	Fano's inequality and the converse to the coding theorem / 204	
8.10	Equality in the converse to the channel coding theorem / 207	
8.11	Hamming codes / 209	
Q 19	Foodback canacity / 212	

xvi CONTENTS

8.13 The joint source channel coding theorem / 215

Summary of Chapter 8 / 218 Problems for Chapter 8 / 220

		Historical notes / 222	
9	Diff	erential Entropy	224
	9.1	Definitions / 224	
	9.2	The AEP for continuous random variables / 225	
	9.3	Relation of differential entropy to discrete entropy / 228	
	9.4	Joint and conditional differential entropy / 229	
	9.5	Relative entropy and mutual information / 231	
	9.6	•••	
	9.7	Differential entropy bound on discrete entropy / 234	
		Summary of Chapter 9 / 236	
		Problems for Chapter 9 / 237	
		Historical notes / 238	
10	The	Gaussian Channel	239
	10.1	The Gaussian channel: definitions / 241	
	10.2	Converse to the coding theorem for Gaussian channels / 245	
	10.3	Band-limited channels / 247	
	10.4	Parallel Gaussian channels / 250	
	10.5	Channels with colored Gaussian noise / 253	
	10.6	Gaussian channels with feedback / 256	
		Summary of Chapter 10 / 262	
		Problems for Chapter 10 / 263	
		Historical notes / 264	
11	Max	imum Entropy and Spectral Estimation	266
	11.1	Maximum entropy distributions / 266	
		Examples / 268	
		An anomalous maximum entropy problem / 270	
		Spectrum estimation / 272	
	11.5	Entropy rates of a Gaussian process / 273	
	11.6	8	
		Summary of Chapter 11 / 277	
		Problems for Chapter 11 / 277	
		Historical notes / 278	

CONTENTS xvii

12	Infor	mation Theory and Statistics	279
	12.1	The method of types / 279	
	12.2	The law of large numbers / 286	
	12.3	Universal source coding / 288	
	12.4	Large deviation theory / 291	
	12.5	Examples of Sanov's theorem / 294	
	12.6	The conditional limit theorem / 297	
	12.7	Hypothesis testing / 304	
	12.8	Stein's lemma / 309	
	12.9	Chernoff bound / 312	
	12.10	Lempel-Ziv coding / 319	
	12.11	Fisher information and the Cramér-Rao inequality / 326	
		Summary of Chapter 12 / 331	
		Problems for Chapter 12 / 333	
		Historical notes / 335	
13	Rate	Distortion Theory	336
	13.1	Quantization / 337	
	13.2	Definitions / 338	
	13.3	Calculation of the rate distortion function / 342	
	13.4	Converse to the rate distortion theorem / 349	
	13.5	Achievability of the rate distortion function / 351	
	13.6	Strongly typical sequences and rate distortion / 358	
	13.7	Characterization of the rate distortion function / 362	
	13.8	Computation of channel capacity and the rate distortion function / 364	
		Summary of Chapter 13 / 367	
		Problems for Chapter 13 / 368	
		Historical notes / 372	
14	Netw	ork Information Theory	374
	14.1	Gaussian multiple user channels / 377	
	14.2	Jointly typical sequences / 384	
	14.3	The multiple access channel / 388	
	14.4	Encoding of correlated sources / 407	
	14.5	Duality between Slepian-Wolf encoding and multiple access channels / 416	
	14.6	The broadcast channel / 418	
	14.7	The relay channel / 428	

CONTENTS xviii

	14.8	Source coding with side information / 432
	14.9	Rate distortion with side information / 438
	14.10	General multiterminal networks / 444
		Summary of Chapter 14 / 450
		Problems for Chapter 14 / 452
		Historical notes / 457
15	Infor	mation Theory and the Stock Market 459
	15.1	The stock market: some definitions / 459
	15.2	Kuhn-Tucker characterization of the log-optimal portfolio / 462
	15.3	Asymptotic optimality of the log-optimal portfolio / 465
	15.4	Side information and the doubling rate / 467
	15.5	Investment in stationary markets / 469
	15.6	Competitive optimality of the log-optimal portfolio / 471
	15.7	The Shannon-McMillan-Breiman theorem / 474
		Summary of Chapter 15 / 479
		Problems for Chapter 15 / 480
		Historical notes / 481
16	Inequ	ualities in Information Theory 482
	16.1	Basic inequalities of information theory / 482
	16.1 16.2	Basic inequalities of information theory / 482 Differential entropy / 485
		-
	16.2	Differential entropy / 485
	16.2 16.3	Differential entropy / 485 Bounds on entropy and relative entropy / 488
	16.2 16.3 16.4 16.5 16.6	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494
	16.2 16.3 16.4 16.5	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497
	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501
	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505
	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505 Overall Summary / 508
	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505 Overall Summary / 508 Problems for Chapter 16 / 509
	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505 Overall Summary / 508
Bib	16.2 16.3 16.4 16.5 16.6 16.7	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505 Overall Summary / 508 Problems for Chapter 16 / 509 Historical notes / 509
	16.2 16.3 16.4 16.5 16.6 16.7 16.8 16.9	Differential entropy / 485 Bounds on entropy and relative entropy / 488 Inequalities for types / 490 Entropy rates of subsets / 490 Entropy and Fisher information / 494 The entropy power inequality and the Brunn-Minkowski inequality / 497 Inequalities for determinants / 501 Inequalities for ratios of determinants / 505 Overall Summary / 508 Problems for Chapter 16 / 509 Historical notes / 509