Contents

Preface xi

Acknowledgments xv

List of Spaces and Norms xvii

I Introductory Topics

Notation 1
Topological Vector Spaces 2
Normed Spaces 3
The Normed Dual 5
Weak Topology and Weak Convergence 6
Compact Sets 6
Uniform Convexity 7
Operators and Imbeddings 8
Spaces of Continuous Functions 9
The Lebesgue Measure in $\mathbb{R}^n$ 12
The Lebesgue Integral 15
Distributions and Weak Derivatives 18

II The Spaces $L^p(\Omega)$

Definition and Basic Properties	22
Completeness of $L^p(\Omega)$	26
Approximation by Continuous Functions, Separability	28
Mollifiers, Approximation by Smooth Functions	29
Precompact Sets in $L^p(\Omega)$	31
The Uniform Convexity of $L^p(\Omega)$	34
The Normed Dual of $L^p(\Omega)$	38

III The Spaces $W^{m,p}(\Omega)$

Definitions and Basic Properties	44
Duality, the Spaces $W^{-m,p'}(\Omega)$	47
Approximation by Smooth Functions on Ω	51
Approximation by Smooth Functions on $\mathbb{R}^n$	53
Approximation by Functions in $C_0^\infty(\Omega)$; (m,p')-Polar Sets	56
Transformation of Coordinates	63

IV Interpolation and Extension Theorems

Geometrical Properties of Domains	65
Interpolation Inequalities for Intermediate Derivatives	70
Interpolation Inequalities Involving Compact Subdomains	79
Extension Theorems	83

V Imbeddings of $W^{m,p}(\Omega)$

The Sobolev Imbedding Theorem	95
Proof of the Imbedding Theorem	99
Traces of Functions in $W^{m,p}(\Omega)$ on the Boundary of Ω	113
$W^{m,p}(\Omega)$ as a Banach Algebra	115
Counterexamples and Nonimbedding Theorems	117
Imbedding Theorems for Domains with Cusps	123
Imbedding Inequalities Involving Weighted Norms	128
Proofs of Theorems 5.35–5.37	138

VI Compact Imbeddings of $W^{m,p}(\Omega)$

The Rellich–Kondrachov Theorem 143
Two Counterexamples 148
Unbounded Domains—Compact Imbeddings of $W_0^{m,p}(\Omega)$ 150
An Equivalent Norm for $W_0^{m,p}(\Omega)$ 158
Unbounded Domains—Decay at Infinity 160
Unbounded Domains—Compact Imbeddings of $W^{m,p}(\Omega)$ 169
Hilbert–Schmidt Imbeddings 173

VII Fractional Order Spaces

Outline 177
The Bochner Integral 178
Semigroups of Operators and the Abstract Cauchy Problem 180
The Trace Spaces of Lions 184
Semigroup Characterization of Trace Spaces 191
Higher-Order Traces 196
The Spaces $W^{s,p}(\Omega)$ 204
An Intrinsic Norm for $W^{s,p}(\Omega)$ 208
Imbedding Theorems 214
Bessel Potentials—The Spaces $L^{s,p}(\Omega)$ 219
Other Fractional Order Spaces 223

VIII Orlicz and Orlicz–Sobolev Spaces

Introduction 227
N-Functions 228
Orlicz Spaces 232
Duality in Orlicz Spaces 237
Separability and Compactness Theorems 240
A Limiting Case of the Sobolev Imbedding Theorem 242
Orlicz–Sobolev Spaces 246
Imbedding Theorems for Orlicz–Sobolev Spaces 247

References 260

Index 265