Contents

Introduction xiii

First Part 1

1. A short scrapbook on classical black holes 3
   - 1.1 Mathematical black holes . . . 3
   - 1.2 Schwarzschild and Kerr black holes . . . 6
     - 1.2.1 The Schwarzschild black hole . . . 6
     - 1.2.2 The Kerr black hole . . . 10
   - 1.3 Bifurcate Killing horizons . . . 15
   - 1.4 Quantum fields and particles . . . 19
     - 1.4.1 The Boulware state . . . 20
     - 1.4.2 The Hartle-Hawking state . . . 21
     - 1.4.3 The Unruh state . . . 22
   - 1.5 BMS group for asymptotically flat spacetimes . . . 23
   - 1.6 Further readings . . . 26

2. The seminal paper 27
   - 2.1 Particle creation by black holes: The computation . . . 31
   - 2.2 Particle creation by black holes: Dependence on the details 45
     - 2.2.1 Nonsymmetrical collapse . . . 45
     - 2.2.2 The spin of the fields . . . 47
     - 2.2.3 Massive fields . . . 48
     - 2.2.4 Angular momentum . . . 49
     - 2.2.5 Electric charge . . . 50
     - 2.2.6 Back reaction . . . 51

2.3 A very short list of further readings 55

3. Thermality of Hawking radiation: From Hartle-Hawking to Israel and Unruh 57

3.1 Hartle-Hawking approach to black hole radiance 58
3.2 Gibbons-Perry analysis for thermality 64
3.3 Thermofield dynamics and Hartle-Hawking-Israel state . . 66
3.3.1 Thermofield Dynamics 66
3.3.2 Israel contribution 68
3.4 Unruh's cornerstone 71

4. The tunneling approach 73

4.1 Damour-Ruffini 74
4.2 Hamilton-Jacobi tunneling method 76
4.2.1 Canonical invariance 78
4.2.2 The null geodesic method 78
4.2.3 The analyticity argument 81
4.2.4 A trick à la Nikishov 83
4.2.5 The 4D case: Role of the transverse coordinates . . 86
4.3 Parikh-Wilczek approach 87

5. The anomaly route to Hawking radiation 89

5.1 Christensen-Fulling way (1977) 89
5.2 Robinson-Wilczek (2005) and Iso-Umetsu-Wilczek (2006) . 93

6. The Euclidean section and Hawking temperature 101

6.1 Local diffeomorphism and extendability 102
6.2 Conical singularity and black hole horizon 104
6.2.1 The 2D case 104
6.2.2 The 4D case 105
6.3 The Hawking temperature from the extendability of the metric at $r = r_+$ 106
6.4 Example: Schwarzschild black hole 108
6.5 The failure of the argument for extremal black holes 108

7. Rigorous aspects of Hawking radiation 111

7.1 Local definiteness principle and local stability 112
7.1.1 The local definiteness 113

7.2 Hawking temperature from local definiteness and stability 115
7.3 Existence of Hartle-Hawking state 120
7.3.1 Almost free double quantum dynamical systems . . 120
7.3.2 The Minkowski vacuum 124
7.3.3 The Hartle-Hawking state 127
7.3.4 The Boulware one particle structure 129
7.4 Hawking temperature for a spacetime with bifurcated Killing horizon 131
7.4.1 Modular inclusion and Hawking temperature . . . 131
7.5 Further readings for black holes of the Kerr-Newman family 133

Second Part **135**

8. The roots of analogue gravity 137
8.1 Experimental black hole evaporation in water 138
8.2 Analogue systems and dispersion: The Gospel according to Unruh 141
8.3 A sample model for dispersive fluids: Essentials 144
8.4 Analogue gravity in BEC 147
8.5 Further readings concerning analogue gravity in presence of dispersion 150

9. Hawking radiation in a non-dispersive nonlinear Kerr dielectric 153
9.1 Classical analysis of the effective spacetime geometry . . . 154
9.1.1 Wave rays geometry in the RIP frame 157
9.1.2 Horizons in effective geometries 159
9.1.3 Null geodesics in dispersionless and in dispersive dielectrics 163
9.1.4 Trapping in dispersive regime 168
9.2 Hawking radiation in a static dielectric black hole 170
9.2.1 The "standard" Hawking prediction 175
9.2.2 The electrodynamics point of view 177
9.3 Effects of optical dispersion: Preliminary heuristics 185
9.3.1 Horizons in dispersive media 188

10. Hawking radiation in a dispersive Kerr dielectric 195

10.1 The relativistic Hopfield model 196
10.1.1 The covariant generalization of the Hopfield model 197
10.1.2 Quantum covariant Hopfield model 199
10.2 Uniformly moving RIP 214
10.3 Hawking radiation in the perturbative formulation 216
10.3.1 The $\varphi\psi$–model 217
10.3.2 Separation of variables 220
10.3.3 Quantization of the fields 221
10.4 An interlude: Semi-analytical and numerical calculations from Maxwell equations in the lab 228
10.4.1 Born approximation 229
10.4.2 Thermality for a Gaussian perturbation 231
10.4.3 A sample of numerical results 233
10.5 Calculation of thermality in the $\phi\psi$ model 234
10.5.1 Determination of the microscopic parameters in terms of the physical ones 236
10.5.2 Near horizon approximation: Solutions of equation (10.232) 237
10.5.3 Steepest descent approximation 238
10.5.4 Convergence regions 241
10.5.5 Decreasing mode inside the black hole $x < 0$ 241
10.5.6 Possible diagrams in the external region $x > 0$ . . . 241
10.5.7 Special configurations and thermality of pair-creation 242
10.5.8 Branch cuts along steepest descent paths 245
10.5.9 Vertical branch cuts 246
10.5.10 Near horizon: A different saddle point approximation 249
10.5.11 WKB solutions 252
10.5.12 A dimensionless parameter for the saddle point approximation 255
10.5.13 A further rescaling and insights for thermality . . . 257
10.5.14 Coalescence of branch points as $\omega \to 0$ 259
10.5.15 Horizons and dispersion 261
10.6 Recapitulation 263
10.7 Further readings 264
10.8 Hawking radiation in a dispersive nonlinear dielectric . . . 265

11. Hawking radiation in the lab 267

11.1 The Como experiment 267
11.2 The Vancouver experiment 276
11.3 The Technion experiment 278

Appendix A Algebraic methods in QFT 281

A.1 Araki-Haag-Kastler algebraic formulation of QFT 281
A.2 Haag-Hugenholz-Winnik formulation of quantum statistical systems 286
A.2.1 Structure of the statistical system 286
A.2.2 The Gibbs states 289
A.2.3 KMS condition and the infinite volume limit . . . 290
A.2.4 The thermal representations 292
A.3 The Tomita-Takesaki theorem 294
A.3.1 Polar decomposition 294
A.3.2 Some simple facts about Von Neumann algebras . 295
A.3.3 The Tomita-Takesaki theorem and the KMS condition 297

Bibliography 299

Index 321