

Contents

1	Probability	1
1.1	The Elements of Probability: $(\Omega, \mathcal{B}, \mu)$	1
1.1.1	Events and Sample Space: (Ω)	1
1.1.2	σ -algebras (\mathcal{B}_Ω) and Measurable Spaces $(\Omega, \mathcal{B}_\Omega)$	3
1.1.3	Set Functions and Measure Space: $(\Omega, \mathcal{B}_\Omega, \mu)$	6
1.1.4	Random Quantities.....	10
1.2	Conditional Probability and Bayes Theorem.....	14
1.2.1	Statistically Independent Events.....	15
1.2.2	Theorem of Total Probability.....	18
1.2.3	Bayes Theorem.....	19
1.3	Distribution Function.....	23
1.3.1	Discrete and Continuous Distribution Functions.....	24
1.3.2	Distributions in More Dimensions.....	28
1.4	Stochastic Characteristics.....	35
1.4.1	Mathematical Expectation.....	35
1.4.2	Moments of a Distribution.....	36
1.4.3	The “Error Propagation Expression”.....	44
1.5	Integral Transforms.....	45
1.5.1	The Fourier Transform.....	45
1.5.2	The Mellin Transform.....	53
1.6	Ordered Samples.....	63
1.7	Limit Theorems and Convergence.....	67
1.7.1	Chebyshev’s Theorem.....	68
1.7.2	Convergence in Probability.....	69
1.7.3	Almost Sure Convergence.....	70
1.7.4	Convergence in Distribution.....	71
1.7.5	Convergence in L_p Norm.....	76
1.7.6	Uniform Convergence.....	77

Appendices	81
References	85
2 Bayesian Inference	87
2.1 Elements of Parametric Inference	88
2.2 Exchangeable Sequences	89
2.3 Predictive Inference	91
2.4 Sufficient Statistics	92
2.5 Exponential Family	94
2.6 Prior Functions	95
2.6.1 Principle of Insufficient Reason	96
2.6.2 Parameters of Position and Scale	97
2.6.3 Covariance Under Reparameterizations	103
2.6.4 Invariance Under a Group of Transformations	109
2.6.5 Conjugated Distributions	115
2.6.6 Probability Matching Priors	119
2.6.7 Reference Analysis	125
2.7 Hierarchical Structures	133
2.8 Priors for Discrete Parameters	135
2.9 Constrains on Parameters and Priors	136
2.10 Decision Problems	137
2.10.1 Hypothesis Testing	139
2.10.2 Point Estimation	145
2.11 Credible Regions	147
2.12 Bayesian (\mathcal{B}) Versus Classical (\mathcal{F}) Philosophy	148
2.13 Some Worked Examples	154
2.13.1 Regression	154
2.13.2 Characterization of a Possible Source of Events	158
2.13.3 Anisotropies of Cosmic Rays	161
References	166
3 Monte Carlo Methods	169
3.1 Pseudo-Random Sequences	170
3.2 Basic Algorithms	171
3.2.1 Inverse Transform	171
3.2.2 Acceptance-Rejection (Hit-Miss; J. Von Neumann 1951)	178
3.2.3 Importance Sampling	183
3.2.4 Decomposition of the Probability Density	185
3.3 Everything at Work	186
3.3.1 The Compton Scattering	186
3.3.2 An Incoming Flux of Particles	192
3.4 Markov Chain Monte Carlo	199
3.4.1 Sampling from Conditionals and Gibbs Sampling	214

3.5	Evaluation of Definite Integrals	218
	References	219
4	Information Theory	221
4.1	Quantification of Information	221
4.2	Expected Information and Entropy	223
4.3	Conditional and Mutual Information	226
4.4	Generalization for Absolute Continuous Random Quantities	228
4.5	Kullback–Leibler Discrepancy and Fisher’s Matrix	229
	4.5.1 Fisher’s Matrix	230
	4.5.2 Asymptotic Behaviour of the Likelihood Function	232
4.6	Some Properties of Information	234
4.7	Geometry and Information	238
	References	244