

Contents

Foreword	xi
Preface	xiii
About the Authors	xvii
About the Cover	xix
Acknowledgments	xxiii
I APPLICATIONS AND PRACTICE	1
1 Introduction and Overview	3
1.1 Mathematics of Data	3
1.2 Data in the World	5
1.3 Mathematical Foundations	9
1.4 Making Data Rigorous	14
1.5 Conclusions, Exercises, and References	16
2 Perspectives on Data	19
2.1 Interrelations	19
2.2 Spreadsheets	20
2.3 Databases	22
2.4 Matrices	26
2.5 Graphs	27
2.6 Map Reduce	29
2.7 Other Perspectives	30
2.8 Conclusions, Exercises, and References	31
3 Dynamic Distributed Dimensional Data Model	37
3.1 Background	37
3.2 Design	38
3.3 Matrix Mathematics	39
3.4 Common SQL, NoSQL, NewSQL Interface	40
3.5 Key-Value Store Database Schema	41
3.6 Data-Independent Analytics	44
3.7 Parallel Performance	49
3.8 Computing on Masked Data	51
3.9 Conclusions, Exercises, and References	53

4	Associative Arrays and Musical Metadata	57
4.1	Data and Metadata	57
4.2	Dense Data	58
4.3	Dense Operations	60
4.4	Sparse Data	62
4.5	Sparse Operations	63
4.6	Conclusions, Exercises, and References	65
5	Associative Arrays and Abstract Art	69
5.1	Visual Abstraction	69
5.2	Minimal Adjacency Array	71
5.3	Symmetric Adjacency Array	73
5.4	Weighted Adjacency Array	75
5.5	Incidence Array	75
5.6	Conclusions, Exercises, and References	78
6	Manipulating Graphs with Matrices	81
6.1	Introduction	81
6.2	Matrix Indices and Values	86
6.3	Composable Graph Operations and Linear Systems	89
6.4	Matrix Graph Operations Overview	96
6.5	Graph Algorithms and Diverse Semirings	105
6.6	Conclusions, Exercises, and References	108
7	Graph Analysis and Machine Learning Systems	115
7.1	Introduction	115
7.2	Data Representation	116
7.3	Graph Construction	118
7.4	Adjacency Array Graph Traversal	120
7.5	Incidence Array Graph Traversal	122
7.6	Vertex Degree Centrality	126
7.7	Edge Degree Centrality	129
7.8	Eigenvector Centrality	129
7.9	Singular Value Decomposition	133
7.10	PageRank	136
7.11	Deep Neural Networks	138
7.12	Conclusions, Exercises, and References	140
II	MATHEMATICAL FOUNDATIONS	145
8	Visualizing the Algebra of Associative Arrays	147
8.1	Associative Array Analogs of Matrix Operations	147
8.2	Abstract Algebra for Computer Scientists and Engineers	150
8.3	Depicting Mathematics	152
8.4	Associative Array Class Diagrams	153

8.5	Set	154
8.6	Semiring	155
8.7	Linear Algebra	158
8.8	Ordered Sets	160
8.9	Boolean Algebra	162
8.10	Associative Array Algebra	164
8.11	Conclusions, Exercises, and References	164
9	Defining the Algebra of Associative Arrays	169
9.1	Operations on Sets	169
9.2	Ordered Sets	175
9.3	Supremum and Infimum	177
9.4	Lattice	181
9.5	The Semirings of Interest	186
9.6	Conclusions, Exercises, and References	189
10	Structural Properties of Associative Arrays	193
10.1	Estimating Structure	193
10.2	Associative Array Formal Definition	194
10.3	Padding Associative Arrays with Zeros	197
10.4	Zero, Null, Zero-Sum-Free	198
10.5	Properties of Matrices and Associative Arrays	199
10.6	Properties of Zero Padding	201
10.7	Support and Size	207
10.8	Image and Rank	208
10.9	Example: Music	209
10.10	Example: Art	211
10.11	Properties of Element-Wise Addition	213
10.12	Properties of Element-Wise Multiplication	217
10.13	Array Multiplication	221
10.14	Closure of Operations between Arrays	228
10.15	Conclusions, Exercises, and References	229
11	Graph Construction and Graphical Patterns	235
11.1	Introduction	235
11.2	Adjacency and Incidence Array Definitions	236
11.3	Adjacency Array Construction	242
11.4	Graph Construction with Different Semirings	250
11.5	Special Arrays and Graphs	255
11.6	Key Ordering	258
11.7	Algebraic Properties	263
11.8	Subobject Properties	264
11.9	Conclusions, Exercises, and References	266

III	LINEAR SYSTEMS	269
12	Survey of Common Transformations	271
12.1	Array Transformations	271
12.2	Identity	274
12.3	Contraction	290
12.4	Stretching	293
12.5	Rotation	297
12.6	Conclusions, Exercises, and References	299
13	Maps and Bases	303
13.1	Semimodules	303
13.2	Linear Maps	307
13.3	Linear Independence and Bases	309
13.4	Existence of Bases	312
13.5	Size of Bases	313
13.6	Semialgebras and the Algebra of Arrays	317
13.7	Conclusions, Exercises, and References	320
14	Linearity of Associative Arrays	323
14.1	The Null Space of Linear Maps	323
14.2	Supremum-Blank Algebras	326
14.3	Max-Blank Structure Theorem	334
14.4	Examples of Supremum-Blank Algebras	338
14.5	Explicit Computations of $x(\mathbf{A}, \mathbf{w})$ for Supremum-Blank Algebras	342
14.6	Conclusions, Exercises, and References	348
15	Eigenvalues and Eigenvectors	351
15.1	Introduction	351
15.2	Quasi-Inverses	353
15.3	Existence of Eigenvalues for Idempotent Multiplication	359
15.4	Strong Dependence and Characteristic Bipolynomial	360
15.5	Eigenanalysis for Irreducible Matrices for Invertible Multiplication	367
15.6	Eigen-Semimodules	373
15.7	Singular Value Decomposition	378
15.8	Conclusions, Exercises, and References	385
16	Higher Dimensions	389
16.1	d -Dimensional Associative Arrays	389
16.2	Key Ordering and Two-Dimensional Projections	392
16.3	Algebraic Properties	398
16.4	Sub-Array Properties	400
16.5	Conclusions, Exercises, and References	402
Appendix: Notation		405
Index		413