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# Units and Fundamental Constants in Physics and Chemistry

Subvolume b  
Fundamental Constants in Physics and Chemistry

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