

# Nonequilibrium Phenomena in Polyatomic Gases

VOLUME 2  
CROSS SECTIONS, SCATTERING,  
AND RAREFIED GASES

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## VOLUME 2

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