Contents

1 Nonlinear Observability and the Observer Design Problem 1
1.1 Observation Problem 1
1.2 Observability and Observer Design for Nonlinear Systems 5
1.2.1 Some Notions of Observability 5
1.2.2 *Observer Design* 7
1.3 Organization of the Book 11
References 12

Part I Normal Forms and Their Observers

2 Introduction 17
References 19

3 State-Affine Normal Forms 21
3.1 Constant Linear Part: Luenberger Design 21
3.1.1 A Hurwitz: Luenberger's Original Form 21
3.1.2 H Linear: $H(\xi, u) = C\xi$ with C Constant 22
3.2 Time-Varying Linear Part: Kalman Design 22
References 26

4 Triangular Forms 29
4.1 Nominal Triangular Form: High-Gain Designs 29
4.1.1 Lipschitz Triangular Form 30
4.1.2 High-Gain Observer For a Non-Lipschitz Triangular Form? 32
4.1.3 Hölder Continuous Triangular Form 34
4.1.4 Continuous Triangular Form 36
4.1.5 Relaxation of Some Assumptions 39
4.2 General Triangular Form: High-Gain-Kalman Design 41
References 43

Part II Transformation into a Normal Form

5 Introduction 49

6 Transformations into State-Affine Normal Forms 53
6.1 Linearization by Output Injection 53
6.1.1 Constant Linear Part 53
6.1.2 Time-Varying Linear Part 56
6.2 Transformation into Hurwitz Form 56
6.2.1 Luenberger Design for Autonomous Systems 57
6.2.2 Luenberger Design for Nonautonomous Systems 59
6.3 Examples 66
6.3.1 Linear Systems with Unknown Parameters 66
6.3.2 State-Affine Systems with Output Injection and Polynomial Output 67
6.3.3 Non-holonomic Vehicle 69
6.3.4 Time-Varying Transformations for Autonomous Systems 70
References 71

7 Transformation Into Triangular Forms 75
7.1 Lipschitz Triangular Form 75
7.1.1 Time-Varying Transformation 76
7.1.2 Stationary Transformation 78
7.2 Continuous Triangular Form 79
7.2.1 Existence of g_i Satisfying (7.6) 81
7.2.2 Lipschitzness of the Triangular Form 87
7.2.3 Back to Example 4.1 91
7.3 General Lipschitz Triangular Form 92
References 94

Part III Expression of the Dynamics of the Observer in the System Coordinates

8 Motivation and Problem Statement 99
8.1 Examples 100
8.1.1 Oscillator with Unknown Frequency 100
8.1.2 Bioreactor 103
8.1.3 General Idea 104
8.2 Problem Statement 105
8.2.1 Starting Point 105
8.2.2 A Sufficient Condition Allowing the Expression of the Observer in the Given x-Coordinates 107
8.3 Direct Construction of the Extended Diffeomorphism T_e? 110
References 113

9 Around Problem 8.1: Augmenting an Injective Immersion into a Diffeomorphism 115
9.1 Submersion Case 116
9.2 The $\tilde{P}[d_\xi, d_x]$ Problem 118
9.3 Wazewski's Theorem 120
References 123

10 Around Problem 8.2: Image Extension of a Diffeomorphism 125
10.1 A Sufficient Condition 125
10.2 Explicit Diffeomorphism Construction for Part (a) of Theorem 10.1 127
10.3 Application: Bioreactor 131
10.4 Conclusion 134
References 135

11 Generalizations and Examples 137
11.1 Modifying T and $\mathcal{O}$ given by Assumption 8.1 137
11.1.1 For Contractibility 138
11.1.2 For a Solvable $\tilde{P}[d_\xi, d_x]$ Problem 139
11.1.3 A Universal Complementation Method 142
11.2 A Global Example: Luenberger Design for the Oscillator 142
11.3 Generalization to a Time-Varying T 147
11.3.1 Partial Theoretical Justification 148
11.3.2 Application to Image-Based Aircraft Landing 149
References 154

Appendix A: Technical Lemmas 157

Appendix B: Lyapunov Analysis for High-Gain Homogeneous Observers 167

Appendix C: Injectivity Analysis for Nonlinear Luenberger Designs 181

Index 187