## Contents

1	Not	ation and Glossary of Fundamental Terms and Symbols	1
2	Fun	damental Concepts	8
	2.1	Boolean Indicator Variables and All Their Functions of One and Two Variables	8
		<ul><li>2.1.1 The Functions of One Variable</li><li>2.1.2 The Functions of Two Variables</li></ul>	9 10
	2.2	Axioms and Elementary Laws of Boolean Algebra	11
		<ul> <li>2.2.1 Axioms</li> <li>2.2.2 Elementary Laws</li> <li>2.2.3 Complete Systems of Operators</li> </ul>	12 13 17
	2.3	Polynomials, Vectors, and Matrices with Boolean Elements	18
		<ul><li>2.3.1 Polynomials with Boolean Coefficients</li><li>2.3.2 Boolean Vector/Matrix Calculus</li></ul>	18 19
	Exe	rcises	20
3	Dia	grams for Boolean Analysis	22
	3.1	Standard Graphical Representation of Boolean Functions	22
	3.2	Binary Decision Diagrams	24
	3.3	Karnaugh Maps	25
	3.4	Switching Network Graphs (Logic Diagrams) and Syntax Diagrams	28
	3.5	Venn Diagrams	30
	3.6	State Transition Graphs	31



	3.7	Communications Graphs	32
	3.8	Reliability Block Diagrams	32
	3.9	Flowcharts	34
	3.10	Petri Nets	35
	Exe	rcises	36
4	Rep	resentations (Forms) and Types of Boolean Functions	37
	4.1	Negation (Complement)	37
	4.2	Special Terms	39
	4.3	Normal Forms and Canonical Normal Forms	43
		<ul><li>4.3.1 Elementary Theory</li><li>4.3.2 Description of Connectivity Properties of Graphs</li></ul>	44 53
	4.4	Disjunctive Normal Forms of Disjoint Terms	59
		<ul> <li>4.4.1 Fundamentals</li> <li>4.4.2 A set-addition algorithm</li> <li>4.4.3 The Shannon Decomposition Algorithm</li> <li>4.4.4 A Binary Decision Tree Algorithm</li> <li>4.4.5 Comparison of DDNF Algorithms</li> </ul>	60 62 70 73 79
	4.5	Boolean Functions with Special Properties	80
	4.6	Recursive Definition of Boolean Functions	87
	4.7	Hazards	90
	Exe	rcises	93
5	Min	imal Disjunctive Normal Forms	95
	5.1	General Considerations	95
	5.2	Finding All Prime Implicants of a Boolean Function	98
		<ul><li>5.2.1 The Quine-McCluskey Procedure</li><li>5.2.2 The Concensus Procedure of Quine</li><li>5.2.3 The Double Negation Procedure of Nelson</li></ul>	99 103 107

	5.3	Minimization	111
		<ul><li>5.3.1 Optimal Choice of Subsets</li><li>5.3.2 Shortest Disjunctive Normal Forms</li><li>5.3.3 Incompletely Specified Boolean Functions</li></ul>	111 113 115
	Exe	rcises	116
6	Boo	lean Difference Calculus	118
	6.1	Exclusiv-Disjunction Form Without Negated Variables	118
	6.2	Concepts of Boolean Differences	123
	6.3	Basic Rules of Boolean Difference Calculus	126
	6.4	Diagnosing Permanent Faults in Switching Networks	128
	Exe	rcises	129
7	Boo	lean Functions Without Boolean Operators	131
	7.1	Fundamental Concepts and Consequences	132
	7.2	Transformation of Boolean Functions of Indicator Variables to Multilinear Form	135
	7.3	Coherence Revisited	141
	Exe	rcises	143
8	Stoc	hastic Theory of Boolean Functions	144
	8.1	Probability of a Binary State	144
	8.2	Probability of the Value 1 of a Boolean Function	147
		<ul> <li>8.2.1 Expected Value of a Boolean Function</li> <li>8.2.2 Probabilities of Arguments (States) in the Homogeneous Markov Model</li> </ul>	148 152
	8.3	Approximate Probability of the Value 1	157
	0.5	<ul><li>8.3.1 Principle of Inclusion-Exclusion</li><li>8.3.2 Approximation with a Given Error Bound</li></ul>	159 158 159
	8.4	Moments of Boolean Functions	1 <b>6</b> 0
	Exe	rcises	162

9	Stoc	hastic Theory of Boolean Indicator Processes	163	
	9.1	Mean Duration of States in the Markov Model	163	
	9.2	Mean Duration of Boolean Functions' Values	166	
	9.3	Mean Frequency of Changes of Functions' Values	172	
	9.4	The Distribution of Residual Life Times	178	
	Exe	cises	181	
10	Som	e Algorithms and Computer Programs for Boolean Analysis	182	
	10.1	Computing Values of a Boolean Function	183	
	10.2	Canonical Representations of a Boolean Function	191	
		10.2.1 The Canonical Disjunctive Normal Form10.2.2 The Canonical Multilinear Polynominal Form	191 193	
	10.3	Probability of a Given Value of a Boolean Function	196	
	10.4	Algorithms for Making the Terms of a Given DNF Disjoint	199	
	10.5	Selected Set Manipulations	207	
	Exe	rcises	216	
11	Арр	endix: Probability Theory Refresher	218	
	11.1	Boolean Algebra of Sets	<b>2</b> 18	
	11.2	Elementary Probability Calculus	220	
	11.3	Random Variables and Random Processes	223	
	11.4	Elementary Renewal Theory	227	
	11.5	Laplace Transform Refresher	230	
	Exe	rcises	233	
Solutions of the exercises for §§ 2 through 11			234	
References				
Subject Index				