## PART 1 MATHEMATICAL PRELIMINARIES: ELEMENTS OF PROBABILITY THEORY AND RANDOM FUNCTIONS

1.1 1.2	Random variables. Statistical moments 3
1.2	Joint probability distributions. Conditional probability. Multivariate normal distributions
1.3	Random functions. Stationarity. Isotropy
1.5 1.4	Differentiation and integration of random functions
1.7	Microscale and integral scale
1.5	Differentiation of random discontinuous functions
1.6	Spectral methods
1.7	Random functions of stationary increments
1.8	Conditional Gaussian probability and interpolation by kriging
1.9	Spatial averages of random functions
1.10	The ergodic hypothesis
1.10	
PART 2	THE LABORATORY SCALE (HOMOGENEOUS MEDIA)
2.1	Introduction
2.2	Geometry of porous media and space averaging
2.2.1	
2.2.2	Space averages and macroscopic variables
2.3	The microscopic equations of flow and transport
2.4	Averaging of derivatives of microscopic variables     61
2.5	Macroscopic variables and macroscopic equations of mass
2.5	and energy conservation
2.5.1	Definition of macroscopic variables
2.5.2	The macroscopic equation of state and of mass conservation
2.5.2	The macroscopic equation of conservation of energy
2.5.4	The macroscopic equation of solute mass conservation
2.6	The macroscopic equations of conservation of momentum
2.7	The constitutive equation of heat transfer (effective heat conductivity)
2.7.1	Definitions and experimental evidence
2.7.2	Theoretical derivation of the constitutive equation and of bounds of effective
2.1.2	conductivity
2.7.3	Evaluation of the effective heat conductivity with the aid of models of porous
2.1.5	media
2.8	The constitutive equation of mass transfer (effective diffusion coefficient) 89
2.9	Darcy's law
2.9.1	Definitions and experimental evidence
2.9.1	Theoretical derivation of Darcy's law
2.9.2	Derivation of permeability with the aid of models
2.9.5	Generalizations of Darcy's law
	Converting diffusive transport (hydrodynamic diamension)
2.10	Convective-diffusive transport (hydrodynamic dispersion)



VП

XII		
2.10.1 2.10.2 2.10.3 2.10.4 2.11 2.11.1 2.11.2 2.11.3 2.12 2.12.1 2.12.2 2.13 2.13	Definitions and experimental evidence	108 114 117 123 125 125 127 128 133 136 138 139 140 142
	A free-surface (water-table, phreatic surface)	144
	Boundary conditions for solute and heat transport	151
PART 3	WATER FLOW AT THE LOCAL (FORMATION) SCALE.	191
3.1	Introduction	157
3.2	The heterogeneous structure of aquifers at the local (formation) scale	160
3.2.1	A few field findings	
3.2.2	Statistical representation of heterogeneous formations	100
	and their classification	163
3.2.3	A few examples of covariances $C_y(r)$	
3.2.4	Statistical properties of the space average $\overline{y}$	
3.2.5	Effect of parameters estimation errors and summarizing	
	comments	176
3.3	General formulation of the direct problem and of the equations of flow	179
3.3.1	General statement of the direct problem	179
3.3.2	A few general observations on the stochastic problem	182
3.4	The effective hydraulic conductivity	187
3.4.1	Steady uniform flow : general statement and absolute bounds	187
3.4.2	Small perturbation, first-order approximation of Kef	189
3.4.3	The self-consistent approach	194
3.4.4	Effective conductivity of a two-phase formation	202
3.4.5	Influence of boundary on effective conductivity	204
3.4.6 3.4.7	Effective conductivity and storativity in unsteady flow	206
5.4.7	through compressible formations	209
3.5	Solutions of the mean flow equations (examples of exact solutions)	209
3.5.1	General	216
3.5.2	Illustration of exact solutions of a few classes of flow	218
3.6	Solutions of the mean flow equations (approximate methods)	228
3.6.1	The method of singularities	229
3.6.2	Linearization of the free-surface condition	230
3.6.3	Flow through layered formations of large conductivity contrast	232
3.6.4	The shallow-water flow approximation (Dupuit-Forcheimer-Boussinesq)	234
3.7	Second-order statistical moments of the flow variables	240
3.7.1	Introduction	240
3.7.2	Steady, uniform in the average, flow in unbounded formations. First-order approximation	242
3.7.3	The effect of nonlinearity of logconductivity variance upon head covariances	250

3.7.3	The effect of nonlinearity of logconductivity variance upon head covariances	
	The effect of boundaries on head covariances	
3.7.5	Specific discharge covariances	•

. . 253 . . 256

3.7.6 3.7.7	The effect of space averaging upon specific discharge variance	
PART 4	SOLUTE TRANSPORT AT THE LOCAL (FORMATION) SCALE.	
4.1	Introduction	265
4.2	A few field findings	266
4.3	The conceptual model	271
4.3.1	General	271
4.3.2	The statistics of fluid particles displacements	
	(the Lagrangian framework)	273
4.3.3	The statistics of particles displacements (the Eulerian framework)	277
4.3.4	The concentration expected value	280
4.3.5	The concentration variance	283
4.3.6	The concentration spatial moments	286
4.4	A few numerical simulations of solute transport in heterogeneous formations	290
4.5	Transport through stratified formations	293
4.5.1	Introduction	293
4.5.2	Steady flow parallel to the bedding	294
4.5.3	Flow tilted with respect to the bedding	
4.6	Transport in formations of three-dimensional heterogeneous structures	
4.6.1	Introduction	
4.6.2	General formulation of the first-order solution	
4.6.3	Time dependent "macrodispersivity" (first-order	
	approximation, high Pe)	314
4.6.4	Asymptotic, large travel time, "macrodispersivity" (first-order solution)	
4.7	Two-dimensional transport and comparison with a field experiment	
4.8	Effects of nonlinearity and unsteadiness.	
4.8.1	Effect of nonlinearity in $\sigma_y^2$ upon transport	
4.8.2	Unsteady and nonuniform mean flow	335
4.9	Transport of reactive solutes. Effect of parameters estimation errors.	
4.9.1	Transport of reactive solutes	339
4.9.2	The effect of parameters estimation errors	
		2.0
PART 5	FLOW AND TRANSPORT AT THE REGIONAL SCALE.	
5.1	Introduction	353
5.2	Analysis of field data and statistical characterization of heterogeneity	
5.2.1	A few field findings	355
5.2.2	Definition of hydraulic properties at the regional scale	
5.2.3	Statistical representation of heterogeneity	
5.3	Mathematical statement of the direct problem	
5.4	Effective properties and the solutions of the equations of mean flow	
5.4.1	Effective transmissivity and storativity (confined flow,	5.0
5	unconditional probability)	370
5.4.2	Effective transmissivity (unconfined flow, unconditional	570
5.4.2	probability)	374
5.4.3	A few solutions of the mean flow equations	377
5.5	Second-order statistical moments of the flow variables. The effect of conditioning.	381
5.5.1	Introduction	381
5.5.2	First-order approximation for steady flow (unconditional probability)	
5.5.3	Solution of the direct problem in steady flow by conditional probability	391
5.5.4	The effect of boundaries on head covariances (unconditional probability)	396
5.5.5	Effects of nonlinearity in $\sigma_y$ and of unsteadiness $\ldots$	402
5.6	The inverse (identification) problem	
5.6.1		
J.U.I		

XIII

5.6.2	Mathematical statement of the identification problem and the	405
	structure of its solution	405
5.6.3	Stochastic identification of aquifer parameters by first-order approximation	<i>A</i> 10
5.6.4		410
5.0.4		
	(Carrera and Neumann, 1986)	424
5.7	Transport at the regional scale	427
5.7.1	General	
5.7.2	Transport from "non-point sources"	430
5.7.3	Transport from "point sources"	
5.8	Modeling transport by travel time approach	
5.8.1	General	438
5.8.2	One-dimensional transport	
5.8.3	Two-dimensional transport	
BIBLIOG	RAPHY	451
SUBJECT	T INDEX	463

XIV