Contents

\mathbf{P}	refac	e						v	
\mathbf{C}	onter	nts						vii	
In	trod	uction						xii	
1	Cat	Categories, Products, Projective and Inductive Limits							
	1.1	\mathbf{T}	he Notion of a Category and Examples					1	
	1.2	Fu	inctors					3	
	1.3	Pr	coducts, Projective Limits and Direct Limits in a Category					4	
		1.3.1	The Projective Limit					4	
		1.3.2	The Yoneda Lemma					6	
		1.3.3	Examples					6	
		1.3.4	Representable Functors					8	
		1.3.5	Direct Limits					9	
	1.4	Ex	cercises	•	•	•		10	
2	Bas	ic Ċon	cepts of Homological Algebra					11	
	2.1	Tl	ne Category \mathbf{Mod}_{Γ} of Γ-modules					11	
	2.2	M	ore Functors					13	
		2.2.1	Invariants, Coinvariants and Exactness					13	
		2.2.2	The First Cohomology Group					15	
		2.2.3	Some Notation					16	
		2.2.4	Exercises					17	
	2.3	Tł	he Derived Functors					19	
		2.3.1	The Simple Principle					20	
		2.3.2	Functoriality					22	
		2.3.3	Other Resolutions					24	
		2.3.4	Injective Resolutions of Short Exact Sequences					24	
			A Fundamental Bemark					26	
			The Cohomology and the Long Exact Sequence					27	
			The Homology of Groups					27	
	24	ጠ	he Functors Ext and Tor				÷	28	
	2.1	241	The Functor Ext		Ĭ	•	•	28	
		2.1.1	The Derived Functor for the Tensor Product	•••	·	·	•	30	
		2.4.3	Exercise	· •		•	•	32	
3	She	aves						35	
-	3.1 Presheaves and Sheaves		resheaves and Sheaves					35	
		3.1.1	What is a Presheaf?					35	
		3.1.2	A Remark about Products and Presheaf				·	36	
		313	What is a Sheaf?		•	•	·	36	
		314	Examples		•	•	•	38	
	29	о.1. 1 М	anifolds as Locally Ringed Spaces	•••	•	•	•	30	
	0.4	111	amond as houng tanged spaces	• •	•	•	٠		

		3.2.1	What Are Manifolds?					39
		3.2.2	Examples and Exercise					41
	3.3	\mathbf{St}	alks and Sheafification					45
		3.3.1	Stalks					45
		3.3.2	The Process of Sheafification of a Presheaf					46
	3.4	T	The Functors f_* and f^*					47
		3.4.1	The Adjunction Formula					48
		342	Extensions and Restrictions	• •	•••	•••	•••	49
	3 5	C	Interiors of Sheaves	•••	•••	•••	•••	49
	0.0	0.		• •	•••	•••	•••	10
4	Coł	omolo	egy of Sheaves					51
	4.1	E	kamples					51
		4.1.1	Sheaves on Riemann surfaces					51
		4.1.2	Cohomology of the Circle					54
	4.2	\mathbf{T}	he Derived Functor					55
		4.2.1	Injective Sheaves and Derived Functors					55
		4.2.2	A Direct Definition of H^1					56
	4.3	Fi	ber Bundles and Non Abelian H^1					59
		4.3.1	Fibrations					59
			Fibre Bundle					59
			Vector Bundles					60
		4.3.2	Non-Abelian H^1					61
		4.3.3	The Reduction of the Structure Group					62
		21010	Orientation					62
			Local Systems					63
			Isomorphism Classes of Local Systems					64
			Principal G-bundels					64
	44	Fi	indamental Properties of the Cohomology of Sheaves					65
		4.4.1	Introduction					65
		442	The Derived Functor to f .	•••	•••	•••	•••	66
		443	Functorial Properties of the Cohomology	•••	• •	• •	•••	68
		4.4.0 A A A	Paracompact Spaces	•••	•••	•••	•••	69
		1.1.1	Applications	•••	•••	• •	•••	75
		4.4.0	Cohomology of Spheres	•••	• •	•••	•••	75
			Orientations	•••	•••	•••	•••	76
			Compact Oriented Surfaces		•••	• •	•••	77
	15	Č	compact Oriented Surfaces	• •	• •	•••	•••	77
	4.0	451	The Čech Complex	•••	•••	• •	•••	77
		4.5.1	The Čech-Complex	•••	• •	• •	•••	- 11
	16	4.0.2 C.	The Cech Resolution of a Shear	• •	•••	• •	•••	01 01
	4.0	ېن ۱۶۱	Introduction	• •	•••	•••	•••	
		4.0.1	The Venticel Eliteration	•••	•••	• •	•••	00
		4.0.2	The Hariagnetal Filtration	•••	•••	•••	•••	00
		4.0.3	The Horizontal Flitration	• •	• •	•••	•••	94
			1 wo Special Cases	• •	• •	•••	•••	90
			Applications of Spectral Sequences	•••	• •	•••	•••	96
		4.6.4	The Derived Category	•••	•••	•••	•••	98
			The Composition Rule	• •	•••	••	•••	101
			Exact Triangles	•••		• •	•••	102

	4.6.5	The Spectral Sequence of a Fibration	103
		Sphere Bundles an Euler Characteristic	104
	4.6.6	Čech Complexes and the Spectral Sequence	105
		A Criterion for Degeneration	107
		An Application to Product Spaces	109
	4.6.7	The Cup Product	111
	4.6.8	Example: Cup Product for the Comology of Tori	115
		A Connection to the Cohomology of Groups	116
	4.6.9	An Excursion into Homotopy Theory	117
4.7	Co	bhomology with Compact Supports	120
	4.7.1	The Definition	120
	4.7.2	An Example for Cohomology with Compact Supports	121
		The Cohomology with Compact Supports for Open Balls	121
		Formulae for Cup Products	123
	4.7.3	The Fundamental Class	125
4.8	Co	homology of Manifolds	126
	4.8.1	Local Systems	126
	4.8.2	Čech Resolutions of Local Systems	127
	4.8.3	Čech Coresolution of Local Systems	129
	4.8.4	Poincaré Duality	132
	4.8.5	The Cohomology in Top Degree and the Homology	138
	4.8.6	Some Remarks on Singular Homology	140
	4.8.7	Cohomology with Compact Support and Embeddings	141
	4.8.8	The Fundamental Class of a Submanifold	143
	4.8.9	Cup Product and Intersections	144
	4.8.10	Compact oriented Surfaces	146
	4.8.11	The Cohomology Ring of $\mathbb{P}^n(\mathbb{C})$	147
4.9	\mathbf{Th}	e Lefschetz Fixed Point Formula	147
	4.9.1	The Euler Characteristic of Manifolds	149
4.10	Th	e de Rham and the Dolbeault Isomorphism	150
	4.10.1	The Cohomology of Flat Bundles on Real Manifolds	150
		The Product Structure on the de Rham Cohomology	153
		The de Rham Isomorphism and the fundamental class	154
	4.10.2	Cohomology of Holomorphic Bundles on Complex Manifolds	156
		The Tangent Bundle	156
		The Bundle $\Omega_M^{p,q}$	158
	4.10.3	Chern Classes	160
		The Line Bundles $\mathcal{O}_{\mathbb{P}^n(\mathbb{C})}(k)$	163
4.11	Ho	dge Theory	164
	4.11.1	Hodge Theory on Real Manifolds	164
	4.11.2	Hodge Theory on Complex Manifolds	169
		Some Linear Algebra	169
		Kähler Manifolds and their Cohomology	172
		The Cohomology of Holomorphic Vector Bundles	175
		Serre Duality	176
	4.11.3	Hodge Theory on Tori	177

5	Cor	npact]	Riemann surfaces and Abelian Varieties	179
	5.1	Co	ompact Riemann Surfaces	179
		5.1.1	Introduction	179
		5.1.2	The Hodge Structure on $H^1(S,\mathbb{C})$	180
		5.1.3	Cohomology of Holomorphic Bundles	185
		5.1.4	The Theorem of Riemann-Roch	191
			On the Picard Group	191
			Exercises	192
			The Theorem of Riemann-Roch	193
		5.1.5	The Algebraic Duality Pairing	194
		5.1.6	Riemann Surfaces of Low Genus	196
		5.1.7	The Algebraicity of Riemann Surfaces	197
			From a Riemann Surface to Function Fields	197
			The reconstruction of S from K	202
			Connection to Algebraic Geometry	209
			Elliptic Curves	211
		5.1.8	Géométrie Analytique et Géométrie Algébrique - GAGA	212
		5.1.9	Comparison of Two Pairings	215
		5.1.10	The Jacobian of a Compact Riemann Surface	217
		5.1.11	The Classical Version of Abel's Theorem	218
		5.1.12	Riemann Period Relations	222
	5.2	Li	ne Bundles on Complex Tori	223
		5.2.1	Construction of Line Bundles	223
			The Poincaré Bundle	229
			Universality of \mathcal{N}	230
		5.2.2	Homomorphisms Between Complex Tori	232
			The Neron Severi group and $Hom(A, A^{\vee})$	234
			The construction of Ψ starting from a line bundle $\ldots \ldots$	235
		5.2.3	The Self Duality of the Jacobian	236
		5.2.4	Ample Line Bundles and the Algebraicity of the Jacobian	237
			The Kodaira Embedding Theorem	237
			The Spaces of Sections	239
		5.2.5	The Siegel Upper Half Space	240
			Elliptic curves with level structure	243
			The end of the excursion	251
		5.2.6	Riemann-Theta Functions	252
		5.2.7	Projective embeddings of abelian varieties	256
		5.2.8	Degeneration of Abelian Varieties	259
			The Case of Genus 1	259
		_	The Algebraic Approach	269
	5.3	То	owards the Algebraic Theory	271
		5.3.1	Introduction	271
			The Algebraic Definition of the Neron-Severi Group	272
			The Algebraic Definition of the Intersection Numbers	273
			The Study of some Special Neron-Severi groups	274
		5.3.2	The Structure of $\operatorname{End}(J)$	278
			The Rosati Involution	278
			A Trace Formula	280

	The Fundamental Class $[S]$ of S under the Abel Map $\ldots \ldots$	284
5.3.3	The Ring of Correspondences	285
5.3.4	An Algebraic Substitute for the Cohomology	286
Bibliography		290
Index		293