## Contents

Pre	Preface				
No	tation		viii		
0	Introduction				
I	Symmetry Groups of Elementary Particles				
1	Lore	ntz Group	8		
	1.1	Euclidean and Minkowski Spaces. Relativistic Notation	8		
	1.2	Homogeneous Lorentz Group	11		
	1.3	Inhomogeneous Lorentz Group–Poincaré Group	14		
	1.4	Complex Lorentz Transformations	15		
2	1.5 Grou 2.1 2.2 2.3	Representations of the Lorentz and Poincaré Groups, Field Functions,         and Physical States         1.5.1       Representation $D^{(0,0)}$ 1.5.2       Representations $D^{(\frac{1}{2},0)}$ and $D^{(0,\frac{1}{2})}$ 1.5.3       Representation $D^{(\frac{1}{2},\frac{1}{2})}$ <b>ips of Internal Symmetries</b> Abelian Unitary Group $U(1)$ Charge Conjugation $C$ 2.3.1 $SU(2)$ Symmetry	16 19 20 23 23 23 24 24 25		
		2.3.2 $SU(3)$ Symmetry	27		
	2.4	Groups of Local Transformations. Gauge Group	29		
3	Prob	lems to Part I	35		
II	Cla	ssical Theory of the Free Fields			
4	Lagr of th	rangian and Hamiltonian Formalisms e Classical Field Theory	39		
	4.1	Variational Principle and Canonical Formalismof Classical Mechanics4.1.1Lagrangian Equations	39 39		





		4.1.2 Canonical Variables. Hamiltonian Equations	41
		4.1.3 Poisson Brackets. Integrals of Motion	42
		4.1.4 Canonical Formalism in the Presence of Constraints	43
	4.2	From Classical to Quantum Mechanics. Primary Quantization	48
	4.3	General Requirements to the Lagrangians of the Field Theory	52
	4.4	Lagrange–Euler Equations	53
	4.5	Noether's Theorem and Dynamic Invariants	54
	4.6	Vector of Energy-Momentum	56
	4.7	Tensors of Angular Momentum and Spin	57
	4.8	Charge and the Vector of Current	59
	4.9	Canonical Variables	60
5	Clas	ssical Theory of Free Scalar Fields	61
	5.1	Klein–Fock–Gordon Equation	61
	5.2	Relativistic Invariance of the Klein–Fock–Gordon Equation	62
	5.3	Solutions of the Klein–Fock–Gordon Equation	64
	5.4	Interpretation of Solutions. Hilbert Space of States	66
	5.5	$\widehat{C}$ , $\widehat{P}$ , and $\widehat{T}$ Transformations	70
		5.5.1 Transformation of Charge Conjugation $\widehat{C}$	70
		5.5.2 Space Reflection $\widehat{P}$	72
		5.5.3 Time Reversal $T$	73
		5.5.4 <i>CPT</i> -Invariance	73
	5.6	Representations of the Lorentz Group in the Space of States	74
	5.7	Lagrangian Formalism of the Scalar Field. Dynamic Invariants	78
6	Spir	nor Field	82
	6.1	Dirac Equation	82
		6.1.1 Construction of the Dirac Equation	82
		6.1.2 Properties of Dirac Matrices. Conjugate Equation	83
	6.2	Relativistic Invariance	86
		6.2.1 Transformation Properties of the Spinor Field	8/
		6.2.2 On Reducible and meducible spinor Representations	91
	63	Solutions of the Dirac Equation	0/
	0.5	6.3.1 Structure of Solutions in the Momentum Space	94
		6.3.2 Classification of Solutions. Helicity	97
		6.3.3 Relations Between Spinors	102
		6.3.4 Wave Functions of the Electron and Positron.	
		Charge Conjugation	104

		6.3.5	$\widehat{C}  \widehat{P}  \widehat{T}$ -Transformation	107
	6.4	Lagran	gian Formalism	110
	6.5	Repres	entations of the Lorentz Group	115
		6.5.1	Hilbert Space of States	115
		6.5.2	Representations of the Lorentz Group in the Space of States	117
	6.6	Applic	ations of the Dirac Equation	118
		6.6.1	Dirac Equation in the Presence of External Fields	118
	6.7	Massle	ss Spinor Field	121
		6.7.1	Two-component Massless Spinor Field	121
		673	Are There Actual Particles Corresponding to the	123
		0.7.5	Massless Spinor Fields? Physical Interpretation	
			of Solutions. Neutrino	123
		6.7.4	Lagrangian and Dynamic Invariants	125
		6.7.5	On the Mass of Neutrino and Majorana Spinors	126
7	Vecto	or Field	ls	128
	7.1	Lagran	gian Formalism	128
	7.2	Repres	entations in the Momentum Space	131
	7.3	Decom	position into the Longitudinal and Transverse Components .	131
	7.4	$\widehat{P}, \widehat{T}, \widehat{C}$	$\widehat{C}$ -Transformations	133
8	Elect	tromag	netic Field	135
	8.1	Maxwe	ell Equations	135
	8.2	Potenti	al of the Electromagnetic Field	136
	8.3	Gradie	nt Transformations and the Lorentz Condition: Transversality	
		Condit	ion	137
	8.4	Lagran	gian Formalism for Electromagnetic Fields	139
	8.5	Transv	ersal, Longitudinal, and Time Components	
		of the l	Electromagnetic Field	141
	8.6	Quantu	am-Mechanical Characteristics of Photons	143
	8.7	$\widehat{C}, \widehat{P}, \widehat{C}$	$\widehat{T}$ -Transformations	146
	8.8	Consis	tency of the Lorentz and Gauge Transformations.	
		Variou	s Types of Gauges	146
9	Equ	ations f	or Fields with Higher Spins	149
	9.1	Fields	with Spin 3/2	149
	9.2	Particle	es with Spin 2	151
10	Prob	lems to	Part II	152

III	Cl	assical Theory of Interacting Fields	
11	Gau	ge Theory of the Electromagnetic Interaction	158
	11.1	Principle of Gauge Invariance in the Maxwell Theory	158
	11.2	Schrödinger Equation and Gradient (Gauge) Invariance	159
	11.3	Gauge Principle as the Dynamical Principle of Interaction between the Electromagnetic and Electron-Positron Fields	161
12	Clas	sical Theory of Yang–Mills Fields	164
	12.1	Gauge Principle and the Lagrangian of the Yang–Mills Fields	164
	12.2	Equations of Motion for the Free Yang–Mills fields	167
	12.3	Yang–Mills Fields for Arbitrary Representations of the Group $SU(N)$	169
13	Mas	ses of Particles and Spontaneous Breaking of Symmetry	171
	13.1	Spontaneous Breaking of Symmetry	172
	13.2	Higgs Mechanism for the Local $U(1)$ Symmetry	174
	13.3	Higgs Mechanism for the Local $SU(2)$ symmetry	176
	13.4	Generation of the Masses of Fermions	179
14	On t	he Construction of the General Lagrangian of Interacting Fields	181
	14.1	Lagrangian of the QCD	183
	14.2	Lagrangian of Weak Interactions	184
	14.3	On the Electroweak Interactions	188
	14.4	On the Lagrangian of Great Unification	189
15	Solu	tions of the Equations for Classical Fields: Solitary Waves,	
	Solit	ons, Instantons	191
16	Prot	blems to Part III	197
IV	Se	cond Quantization of Fields	
17	Axio	oms and General Principles of Quantization	201
	17.1	Why Do We Need the Procedure of Second Quantization? Operator	
		Nature of the Field Functions	201
	17.2	Schrödinger, Heisenberg, and Interaction Pictures	202
	17.3	Axioms of Quantization	204
	17.4	Relativistic Heisenberg Equation for Quantized Fields	213 214 215

Cont	ents
------	------

	17.5	Physical Content of Positive- and Negative-Frequency Solutions of	
		Equations for Free-Field Operators	217
18	Qua	ntization of the Free Scalar Field	218
	18.1	Commutation Relations. Commutator Functions	218
	18.2	Complex Scalar Field	220
	18.3	Operator Relations for Dynamic Invariants	221
19	Qua	ntization of the Free Spinor Field	222
	19.1	Commutator Functions of Fermi Fields	222
	19.2	Dynamic Invariants of a Free Spinor Field	224
20	Qua	ntization of the Vector and Electromagnetic Fields. Specific	
	Feat	ures of the Quantization of Gauge Fields	225
	20.1	Quantization of the Complex Vector Field	225
	20.2	Quantization of an Electromagnetic Field	229
		20.2.1 Specific Features and Difficulties of the Quantization of an	220
		20.2.2 Gupta–Bleuler Formalism	229
		20.2.3 Canonical Method of Quantization	236
	20.3	On the Quantization of Gauge Fields	238
21	CP1	. Spin and Statistics	240
	21.1	The Transformation of Charge Conjugation	241
	21.2	The Transformation of Space Reflection	242
	21.3	The Transformation of Time Reversal	243
	21.4	<i>CPT</i> -Theorem and the Connection of Spin and Statistics	246
	21.5	Proof of the Pauli Theorem	248
22	Rep	resentations of Commutation and Anticommutation Relations	250
	22.1	General Structure of the Fock Space	250
	22.2	Representations of Commutation Relations for a Free Real	
		Scalar Field	252
		22.2.1 The Fock Space of Free Scalar Bosons	252
		22.2.2 Operators of Creation and Annihilation in the Fock Space.	252
		22.2.3 Vacuum State of Free Particle. Cyclicity of Vacuum Set of	232
		Exponential Vectors	256
		22.2.4 Construction of Representations of Commutation Relations	
		for a Complex Scalar Field	259

		22.2.5	Construction of Representations of Commutation Relations in the Configuration Space. Relativistic Invariance of a Free Field	260
	22.3	Repres 22.3.1	entation of Anticommutation Relations of Spinor Fields Representation of Anticommutation Relations of the Operators of Creation and Annihilation of Fermions and	262
		22.3.2	Antifermions          Representation of Anticommutation Relations in the         Configuration Space	26: 26:
	22.4	Space	of States of a Free Electromagnetic Field	26
	22.5	Space	of Occupation Numbers	27
23	Gree	en Func	tions	27
	23.1	Green	Functions of the Scalar Field	27
	23.2	The G	een Functions of Spinor, Vector, and Electromagnetic Fields .	27
	23.3	Time-(	Ordered Product and Green Functions	27
	23.4	Wick 7	Theorems	27
		23.4.1	Wick Theorem for Normal Products	27
		23.4.2 23.4.3	Generalized Wick Theorem	28 28
	23.5	Operat	ion of Multiplication and the Regularization of Distributions .	28
	23.6	N-Poi	nt Green Functions of Free Fields	28
24	Prot	olems to	Part IV	28
v	Qu	antum	Theory of Interacting Fields. General Problems	
25	Con	structio	on of Quantum Interacting Fields and Problems	
	of T	his Con	struction	29
	25.1	Forma	l Construction of a Quantum Field	29
	25.2	Mathe	matical Problems of Construction of a Quantum	
		Interac	ting Field	29
26	Scat	tering [	Theory. Scattering Matrix	29
	26.1	Quanti	um Description of Scattering. Definition of Scattering	•
		Operat	or	29
	26.2	Forma	Construction of the Scattering Operator by the Method	20
	26.2	or Pert	urbanon meory	20
	26.3	Main I	Normal Form of the Operator S	- 30 - 30
		26.3.1	Invariance of the Scattering Matrix under Lorentz	50
			Transformations and Transformations of Charge Conjugation	30

		26.3.3 Unitarity of the Scattering Operator	310
		26.3.4 Law of Conservation of Energy26.3.5 Matrix Elements of the S-Operator and the Scattering	311
		Amplitude	313
	26.4	Feynman Diagrams	316
		<ul> <li>26.4.1 Feynman Diagrams for the S-Operator</li> <li>26.4.2 Feynman Diagrams for Coefficient Functions</li> </ul>	317
		of the S-Operator	317
	26 5	Effective Cross Sections and Sectoring Matrix	277
	20.5	26.5.1. Classical Picture	323
		26.5.2 Quantum Picture	325
27	Equa	ations for Coefficient Functions of the S-Matrix	327
	27.1	Creation and Annihilation Operators of External Lines of Feynman	
		Diagrams	328
	27.2	Equations of the Resolvent Type	331
	27.3	Equations of the Evolution Type	334
28	Gre	en Functions and Scattering Matrix	336
	28.1	Green Functions and the S-Matrix in the Interaction Picture	336
	28.2	Schwinger Equation for Green Functions	338
	28.3	On the Relationship between the Green Functions and the Coefficient	
		Functions of the Scattering S-Operator	341
	28.4	Equations for Green Functions in Terms of Functional Derivatives .	342
	28.5	Equations for Truncated Green Functions	344
	28.6	Equations for One-Particle Irreducible Green Functions.	347
	28.7	Spectral Representation of the 2-Point Green Function	
		(Källén–Lehmann Representation)	353
29	On	Renormalization in Perturbation Theory	358
	29.1	Primitively-Divergent Diagrams. Separation of Divergences by the Pauli–Villars Method	358
	29.2	Degree of Divergence of Feynman Diagram	365
	29.3	Elimination of Divergences by the Method of Bogoliubov–Parasiuk <i>R</i> -Operation	368
	29 <i>1</i>	<i>R</i> -Operation and Counterterms of a Lagrangian	374
	20.4	Classification of Interactions: Renormalizable and	2,1
	27.5	Nonrenormalizable Theories	379

	29.6	Relationship between Counterterms and the Renormalization of Main Constants of the Theory	380
	29.7	Equivalent Types of Renormalizations	387
30	Metł	nod of Functional (Path) Integrals in Quantized Field Theory	391
	30.1	Notion of Path Integration and Main Formulas	392
	30.2	Formalism of Feynman Integrals (Path Integrals) in Quantum	
		Mechanics	400
	30.3	Formalism of Feynman Integrals for Systems with Constraints	405
	30.4	Path Integral Representation for Scalar Fields	409
	30.5	Path Integral Representation for Fermi Fields	411
31	Prob	lems to Part V	416
VI	Ах	tiomatic and Euclidean Field Theories	
32	Wigl	ntman Axiomatics	423
	32.1	Wightman Axioms for Real Scalar Fields	423
	32.2	Wightman Functions and Their Properties	425
	32.3	Reconstruction Theorem	427
33	Othe	er Axiomatic Approaches	429
	33.1	Haag–Ruelle Scattering Theory (HRST)	429
	33.2	Lehmann–Symanzik–Zimmermann Axiomatics	432
	33.3	Bogoliubov–Medvedev–Polivanov (BMP) Axiomatic Approach	435
34	Eucl	idean Field Theory	439
	34.1	Analytic Continuation of Feynman Amplitudes	440
	34.2	Operators of Free Euclidean Fields	442
		34.2.1 Real scalar field	442
	212	S4.2.2 Euclidean Fermi fields	444
	34.5	Euclidean Green Functions of Interacting Fields	447
75	.т.т Т.т.т	A view sties	152
35	Euc	Idean Axiomatics	455
	35.1	Analytic Continuation of Generalized Wignithan Functions	433
	33.2	Euclidean Green Functions. Usterwalder–Schrader Axioms	433
	33.3	Reconstruction of the wightman i neory	457
- 36	Prol	blems to Part VI	460

## VII Quantum Theory of Gauge Fields

37	Qua	ntum El	lectrodynamics (QED)	465
	37.1	Quantiz 37.1.1	zation of Interacting Electromagnetic Fields	466
		37.1.2	Electromagnetic Fields	466
			in the Coulomb Gauge	467
		37.1.3	Photon Propagator and Gauge Conditions	469
	37.2	S-Matr 37.2.1 37.2.2	Perturbation Theory. Feynman Diagrams	471 471
			and Annihilation Operators of Lines of Feynman Diagrams .	474
		37.2.3 37.2.4	Furry Theorem       Gauge Invariance for Coefficient Functions of the         Gauge Invariance for Coefficient Functions of the	476
			S-Operator	479
	37.3	Equation	ons for Green Functions and Coefficient Functions of the	400
		37.3.1 37.3.2	Schwinger Equation Solution System of Equations for Self-Energy and Vertex Parts	480 480
			of Green Functions	482
	37.4	Diverge	ences in QED and Methods for Their Elimination	485
		37.4.1	Primitively-Divergent Diagrams and Their Regularization .	485
		37.4.2	Mass and Charge Renormalization of Electron (Positron)	490
	37.5	Spectra	al Representations of 2-Point Green Functions	493
38	Qua	ntizatio	n of Gauge Fields	498
	38.1	Path In	tegral for Green Functions in QED (Coulomb Gauge)	499
	38.2	Covaria	ant Gauges: Popov–Faddeev–de Witt Method	502
	38.3	Covaria	ant Quantization of Electromagnetic Interaction	506
		38.3.1	Connection between Different Gauges	507
		38.3.2	Ward Identity	508
	38.4	Quantiz	zation of Yang-Mills Fields Interacting with Matter Fields	510
	38.5	Faddee	ev–Popov Ghosts	514
	38.6	BRST-	Invariance	516
39	Stan	dard M	odels of Interactions	521
	39.1	Renorm	nalization of Gauge Theories	521
	39.2	On the 39.2.1	Masses of Gluons and Spontaneous Symmetry Breakdown . Connection of the Radius of Interaction and the Mass of	530
			Exchange Bosons	530

39.2.2 Are Theories with Nonzero Mass of	f Exchange Bosons
Renormalizable?	
39.2.3 Spontaneous Breakdown of the $U($	1)-Symmetry 533
39.2.4 Spontaneous Breakdown of the Lo	cal $SU(N)$ -Symmetry . 535
39.3 Models of Interactions of Elementary Parti	cles
39.3.1 Strong Interaction. Model of QCD	
39.3.2 Weak and Electroweak Interactions	539
40 Problems to Part VII	542
Appendix Hints for the Solution of Problems	544
Bibliography	549
Index	562

.