

Contents

Preface	v
Introduction	1
I Preliminary Material	
1 What is a Singular Trace?	15
1.1 Compact Operators	15
1.2 Calkin Correspondence	22
1.3 Examples of Traces	28
1.3.1 The Canonical Trace	29
1.3.2 The Dixmier Trace	29
1.3.3 Lidskii Formulation of Traces	33
1.4 Notes	34
2 Preliminaries on Symmetric Operator Spaces	38
2.1 Von Neumann Algebras	38
2.2 Semifinite Normal Traces	42
2.3 Generalized Singular Value Function	46
2.4 Calkin Correspondence in the Semifinite Setting	53
2.5 Symmetric Operator Spaces	56
2.6 Examples of Symmetric Operator Spaces	59
2.7 Traces on Symmetric Operator Spaces	68
2.8 Notes	70
II General Theory	
3 Symmetric Operator Spaces	79
3.1 Introduction	79
3.2 Submajorization in the Finite-dimensional Setting	80
3.3 Hardy–Littlewood(–Polya) Submajorization	83
3.4 Uniform Submajorization	88
3.5 Symmetric Operator Spaces from Symmetric Function Spaces	97
3.6 Symmetric Function Spaces from Symmetric Sequence Spaces	101
3.7 Notes	104

4	Symmetric Functionals	107
4.1	Introduction	107
4.2	Jordan Decomposition of Symmetric Functionals	109
4.3	Lattice Structure on the Set of Symmetric Functionals	114
4.4	Lifting of Symmetric Functionals	117
4.5	Figiel–Kalton Theorem	120
4.6	Existence of Symmetric Functionals	123
4.7	Existence of Fully Symmetric Functionals	130
4.8	The Sets of Symmetric and Fully Symmetric Functionals are Different	133
4.9	Symmetric Functionals on Symmetric Operator Spaces	142
4.10	How Large is the Set of Symmetric Functionals?	146
4.11	Notes	152
5	Commutator Subspace	153
5.1	Introduction	153
5.2	Normal Operators in the Commutator Subspace	155
5.3	Normal Operators in the Closed Commutator Subspace	162
5.4	Subharmonic Functions on Matrix Algebras	168
5.5	Quasi-nilpotent Operators Belong to the Commutator Subspace	173
5.6	Description of the Commutator Subspace	182
5.7	Commutator Subspace of the Weak Ideal $\mathcal{L}_{1,\infty}$	187
5.8	Notes	192
6	Dixmier Traces	194
6.1	Introduction	194
6.2	Extended Limits	196
6.3	Dixmier Traces on Lorentz Ideals	198
6.4	Fully Symmetric Functionals on Lorentz Ideals are Dixmier Traces	203
6.5	Dixmier Traces on Fully Symmetric Ideals of $\mathcal{L}(H)$	206
6.6	Relatively Normal Functionals	209
6.7	Wodzicki Representation of Dixmier Traces	214
6.8	Notes	217

III Traces on Lorentz Ideals

7	Lidskii Formulas for Dixmier Traces on Lorentz Ideals	225
7.1	Introduction	225
7.2	Distribution Formulas for Dixmier Traces	226
7.3	Lidskii Formulas for Dixmier Traces	232
7.4	Special Cases and Counterexamples	235
7.5	Diagonal Formulas for Dixmier Traces Fail	241
7.6	Notes	242
8	Heat Kernel Formulas and ζ-function Residues	244
8.1	Introduction	244
8.2	Heat Kernel Functionals	246
8.3	Fully Symmetric Functionals are Heat Kernel Functionals	252
8.4	Generalized Heat Kernel Functionals	256
8.5	Reduction of Generalized Heat Kernel Functionals	258
8.6	ζ -function Residues	263
8.7	Not Every Dixmier Trace is a ζ -function Residue	268
8.8	Notes	271
9	Measurability in Lorentz Ideals	272
9.1	Introduction	272
9.2	Positive Dixmier Measurable Operators in Lorentz Ideals	273
9.3	Positive Dixmier Measurable Operators in $\mathcal{M}_{1,\infty}$	276
9.4	C -invariant Extended Limits	281
9.5	Positive M -measurable Operators	287
9.6	Additional Invariance of Dixmier Traces	291
9.7	Measurable Operators in $\mathcal{L}_{1,\infty}$	297
9.8	Notes	300

IV Applications to Noncommutative Geometry

10	Preliminaries to the Applications	311
10.1	Summary of Traces on $\mathcal{L}_{1,\infty}$ and $\mathcal{M}_{1,\infty}$	311
10.2	Pseudo-differential Operators and the Noncommutative Residue	317
10.3	Pseudo-differential Operators on Manifolds	330
10.4	Notes	334

11 Trace Theorems	336
11.1 Introduction	336
11.2 Modulated Operators	339
11.3 Laplacian Modulated Operators and Extension of the Noncommutative Residue	345
11.4 Eigenvalues of Laplacian Modulated Operators	355
11.5 Trace Theorem on \mathbb{R}^d	359
11.6 Trace Theorem on Closed Riemannian Manifolds	362
11.7 Integration of Functions	372
11.8 Notes	380
12 Residues and Integrals in Noncommutative Geometry	382
12.1 Introduction	382
12.2 The Noncommutative Residue in Noncommutative Geometry	385
12.3 The Integral in Noncommutative Geometry	390
12.4 Example of Isospectral Deformations	396
12.5 Example of the Noncommutative Torus	405
12.6 Classical Limits	411
12.7 Notes	416
A Operator Results	420
A.1 Matrix Results	420
A.2 Operator Inequalities	422
Bibliography	429
Index	445