Contents

Preface ix							
Preface to the Second Edition xiii							
I	Ele	ments of Mathematical Logic	1				
1	Synt	ax of First-Order Languages	3				
	1.1	Symbols of first-order languages	6				
	1.2	Terms	8				
	1.3	Logical formulas	9				
	1.4	Free variables and substitutions	11				
	1.5	Gödel terms of formulas	15				
	1.6	Proof by structural induction	17				
2	Mod	els of First-Order Languages	23				
	2.1	Domains and interpretations	26				
	2.2	Assignments and models	28				
	2.3	Semantics of terms	28				
	2.4	Semantics of logical connective symbols	29				
	2.5	Semantics of formulas	31				
	2.6	Satisfiability and validity	34				
	2.7	Valid formulas with \leftrightarrow	35				
	2.8	Herbrand universe, Herbrand base and Herbrand set	37				
	2.9	Herbrand model	39				
	2.10	Herbrand model with variables*	43				
	2.11	Substitution lemma	45				
	2.12	Theorem of isomorphism	50				
3	Forn	nal Inference Systems	55				
	3.1	G inference system	59				
	3.2	Inference trees, proof trees and provable sequents	62				
	3.3	Soundness of the G inference system	68				
	3.4	Completeness of the G inference system	72				
	3.5	Consistency and compactness	75				
	3.6	Some commonly-used inference rules	77				
	3.7	Proof theory and model theory	79				



4	Com	iputability & Representability	83
	4.1	Formal theory	84
	4.2	Theory of elementary arithmetic	86
	4.3	$P-kernel on \mathbb{N} $	89
	4.4	Church-Turing thesis	92
	4.5	The problem of representability	94
	4.6	The states of P-kernel	95
	4.7	Operational calculus of P-kernel	97
	4.8	Representations of statements	99
		4.8.1 The representation of the assignment statement	100
		4.8.2 Representation of the sequential statement	102
		4.8.3 Representation of the if statement	103
		4.8.4 Representation of the while statement	104
		4.8.5 Representation of the call statement	112
	4.9	Representability theorem	114
5	Göd	el Theorems	117
	5.1	A self-referential proposition	118
	5.2	Decidable sets	120
	5.3	Fixed point equation in Π	124
	5.4	Gödel's incompleteness theorem	127
	5.5	Gödel's consistency theorem	129
	5.6	Halting problem	132
			152
			152
	_		152
П	Lo	ogical Framework of Scientific Discovery	137
П 6	La	ogical Framework of Scientific Discovery	132 137 139
П 6	L o Seq 6.1	ogical Framework of Scientific Discovery uences of Formal Theories Two examples	137 139 140
П 6	Lo Sequ 6.1 6.2	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories	137 139 140 144
П 6	Lo Sequ 6.1 6.2 6.3	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes	137 139 140 144 148
П 6	Lo Sequ 6.1 6.2 6.3 6.4	ogical Framework of Scientific Discovery cences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences	137 139 140 144 148 150
П 6	Lo Sequ 6.1 6.2 6.3 6.4 6.5	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences	137 139 140 144 148 150 153
П 6	Lo Sequ 6.1 6.2 6.3 6.4 6.5 6.6	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences Forcing sequences	137 139 140 144 148 150 153 156
П 6	Lo Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences Forcing sequences Justification for proschemes	137 139 140 144 148 150 153 156 159
П 6	Lcc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences Forcing sequences Justification for proschemes	137 139 140 144 148 150 153 156 159
П 6 7	Lcc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Revi	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes Proschemes Default extension sequences Forcing sequences Justification for proschemes	137 139 140 144 148 150 153 156 159 161
П 6 7	Loc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Proschemes Default extension sequences Forcing sequences Justification for proschemes Sision Calculus Necessary antecedents of formal consequences	137 139 140 144 148 150 153 156 159 161 162
П 6 7	Loc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1 7.2	ogical Framework of Scientific Discovery uences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences Forcing sequences Justification for proschemes Sision Calculus Necessary antecedents of formal consequences New conjectures and new axioms	137 139 140 144 148 150 153 156 159 161 162 166
П 6 7	Lo Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1 7.2 7.3	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes Resolvent sequences Default extension sequences Forcing sequences Justification for proschemes Sision Calculus Necessary antecedents of formal consequences New conjectures and new axioms Refutation by facts and maximal contraction	137 139 140 144 148 150 153 156 159 161 162 166 166
П 6 7	Lo Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1 7.2 7.3 7.4	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes Proschemes Default extension sequences Forcing sequences Justification for proschemes Sision Calculus Necessary antecedents of formal consequences New conjectures and new axioms Refutation by facts and maximal contraction	137 139 140 144 148 150 153 156 159 161 162 166 166 166
П 6 7	Loc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1 7.2 7.3 7.4 7.5	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes Proschemes Default extension sequences Default extension sequences Forcing sequences Justification for proschemes Necessary antecedents of formal consequences New conjectures and new axioms Refutation by facts and maximal contraction Reachability, soundness and completeness of R -calculus	137 139 140 144 148 150 153 156 159 161 162 166 166 166 169 178
П 6 7	Loc Sequ 6.1 6.2 6.3 6.4 6.5 6.6 6.7 Rev 7.1 7.2 7.3 7.4 7.5 7.6	ogical Framework of Scientific Discovery nences of Formal Theories Two examples Sequences of formal theories Proschemes Proschemes Default extension sequences Default extension sequences Forcing sequences Justification for proschemes ision Calculus Necessary antecedents of formal consequences New conjectures and new axioms Refutation by facts and maximal contraction Reachability, soundness and completeness of R -calculus On the logical framework of scientific discovery	137 139 140 144 148 150 153 156 159 161 162 166 166 169 178 184

Contents

		7.6.2 Darwin's theory of evolution	187				
	7.7	The R -calculus without cut	191				
		7.7.1 The R -calculus without cut	194				
		7.7.2 Examples	197				
		7.7.3 Reachability, Soundness and Completeness	200				
	7.8	Operational semantics for program debugging	201				
		7.8.1 Basic ideas of program debugging	202				
		7.8.2 Structural operational semantics for the tracing procedure	204				
		7.8.3 Operational semantics for the locating procedure	207				
8	Vers	ion Sequences	217				
	8.1	Versions and version sequences	219				
	8.2	The proscheme OPEN	220				
	8.3	Convergence of the proscheme	224				
	8.4	Commutativity of the proscheme	226				
	8.5	Independence of the proscheme	228				
	8.6	Reliable proschemes	230				
9	Indu	active Inference	235				
	9.1	Ground terms, basic sentences, and basic instances	238				
	9.2	Inductive inference system A	240				
	9.3	Inductive versions and inductive process	245				
	9.4	The proscheme GUINA	245				
	9.5	Convergence of the proscheme GUINA	252				
	9.6	Commutativity of the proscheme GUINA	254				
10	Meta	a-Language Environments	257				
	10.1	Three language environments	258				
	10.2	Basic principles of meta-language environments	262				
	10.3	Axiomatization	266				
	10.4	Formal methods	268				
	10.5	Workflow of scientific research	274				
Aŗ	Appendix 1 Sets and Maps						
Aŗ	Appendix 2 Proof of the Representability Theorem						
Bi	Bibliography						
In	Index						