## Contents

Preface to the Third English Edition						
Preface to the Fourth Russian Edition						
4	Sequ	ences and Sums of Independent Random Variables	1			
	1	Zero–One Laws	1			
	2	Convergence of Series	6			
	3	Strong Law of Large Numbers	12			
	4	Law of the Iterated Logarithm	22			
	5	Probabilities of Large Deviations	27			
5	Stati	ionary (Strict Sense) Random Sequences				
	and	Ergodic Theory	33			
	1	Stationary (Strict Sense) Random Sequences: Measure-Preserving				
		Transformations	33			
	2	Ergodicity and Mixing	37			
	3	Ergodic Theorems	39			
6	Stati	ionary (Wide Sense) Random Sequences: L <sup>2</sup> -Theory	47			
	1	Spectral Representation of the Covariance Function	47			
	2	Orthogonal Stochastic Measures and Stochastic Integrals	56			
	3	Spectral Representation of Stationary (Wide Sense) Sequences	61			
	4	Statistical Estimation of Covariance Function				
		and Spectral Density	71			
	5	Wold's Expansion	78			
	6	Extrapolation, Interpolation, and Filtering	85			
	7	The Kalman-Bucy Filter and Its Generalizations	95			
7	Mar	tingales	107			
	1	Definitions of Martingales and Related Concepts	107			
	2	Preservation of Martingale Property Under a Random				
		Time Change	119			

	3	Fundamental Inequalities	132	
	4	General Theorems on Convergence of Submartingales		
		and Martingales	148	
	5	Sets of Convergence of Submartingales and Martingales	156	
	6	Absolute Continuity and Singularity of Probability Distributions		
		on a Measurable Space with Filtration	164	
	7	Asymptotics of the Probability of the Outcome of a Random Walk		
		with Curvilinear Boundary	178	
	8	Central Limit Theorem for Sums of Dependent		
		Random Variables	183	
	9	Discrete Version of Itô's Formula	197	
	10	Application of Martingale Methods to Calculation of Probability		
		of Ruin in Insurance	202	
	11	Fundamental Theorems of Stochastic Financial Mathematics: The		
		Martingale Characterization of the Absence of Arbitrage	207	
	12	Hedging in Arbitrage-Free Models	220	
	13	Optimal Stopping Problems: Martingale Approach	228	
8	Mar	kov Chains	237	
	1	Definitions and Basic Properties	237	
	2	Generalized Markov and Strong Markov Properties	249	
	3	Limiting, Ergodic, and Stationary Probability Distributions		
		for Markov Chains	256	
	4	Classification of States of Markov Chains in Terms of Algebraic		
		Properties of Matrices of Transition Probabilities	259	
	5	Classification of States of Markov Chains in Terms of Asymptotic		
		Properties of Transition Probabilities	265	
	6	Limiting, Stationary, and Ergodic Distributions for Countable		
		Markov Chains	277	
	7	Limiting, Stationary, and Ergodic Distributions for Finite Markov		
		Chains	283	
	8	Simple Random Walk as a Markov Chain	284	
	9	Optimal Stopping Problems for Markov Chains	296	
Dev	elopr	nent of Mathematical Theory of Probability:		
His	torica	l Review	313	
His	torica	ll and Bibliographical Notes (Chaps. 48)	333	
Ref	erenc	es	339	
Index				