

Contents

1	Introduction and Qualitative Theory	1
1.1	A Brief and Informal Introduction to Homogenization	1
1.2	The Subadditive Quantity μ and Its Basic Properties	5
1.3	Convergence of the Subadditive Quantity	10
1.4	Weak Convergence of Gradients and Fluxes	17
1.5	Homogenization of the Dirichlet Problem	28
2	Convergence of the Subadditive Quantities	37
2.1	The Dual Subadditive Quantity μ^*	38
2.2	Quantitative Convergence of the Subadditive Quantities	48
2.3	Quantitative Homogenization of the Dirichlet Problem	62
3	Regularity on Large Scales	67
3.1	Brief Review of Classical Elliptic Regularity	68
3.2	A $C^{0,1}$ -Type Estimate	74
3.3	Higher-Order Regularity Theory and Liouville Theorems	82
3.4	The First-Order Correctors	98
3.5	Boundary Regularity	106
3.6	Optimality of the Regularity Theory	117
4	Quantitative Description of First-Order Correctors	123
4.1	The Energy Quantity J_1 and Its Basic Properties	125
4.2	The Additive Structure of J_1	131
4.3	Improvement of Additivity	134
4.4	Localization Estimates	149
4.5	Fluctuation Estimates	163
4.6	Corrector Estimates in Weak Norms	172
4.7	Corrector Oscillation Estimates in Two Dimensions	180

5	Scaling Limits of First-Order Correctors	191
5.1	White Noise and the Gaussian Free Field	191
5.2	Explicit Constructions of Random Fields	200
5.3	Heuristic Derivation of the Scaling Limit	215
5.4	A Central Limit Theorem for J_1	217
5.5	Convergence of the Correctors to a Gaussian Free Field	234
6	Quantitative Two-Scale Expansions	243
6.1	The Flux Correctors	244
6.2	Quantitative Two-Scale Expansions Without Boundaries	250
6.3	Two-Scale Expansions for the Dirichlet Problem	259
6.4	Boundary Layer Estimates	266
7	Calderón–Zygmund Gradient L^p Estimates	275
7.1	Interior Calderón–Zygmund Estimates	275
7.2	Global Calderón–Zygmund Estimates	287
7.3	$W^{1,p}$ -Type Estimates for the Two-Scale Expansion Error	293
8	Estimates for Parabolic Problems	301
8.1	Function Spaces and Some Basic Estimates	302
8.2	Homogenization of the Cauchy–Dirichlet Problem	307
8.3	A Parabolic $C^{0,1}$ -Type Estimate	314
8.4	Parabolic Higher Regularity and Liouville Theorems	319
8.5	Decay of the Green Functions and Their Gradients	320
8.6	Homogenization of the Green Functions	332
9	Decay of the Parabolic Semigroup	347
9.1	Optimal Decay of the Parabolic Semigroup	349
9.2	Homogenization of the Green Functions: Optimal Scaling	369
10	Linear Equations with Nonsymmetric Coefficients	391
10.1	Variational Formulation of General Linear Equations	392
10.2	The Double-Variable Subadditive Quantities	397
10.3	Convergence of the Subadditive Quantities	407
10.4	Quantitative Homogenization of the Dirichlet Problem	412
11	Nonlinear Equations	417
11.1	Assumptions and Preliminaries	417
11.2	Subadditive Quantities and Basic Properties	420
11.3	Convergence of the Subadditive Quantities	428
11.4	Quantitative Homogenization of the Dirichlet Problem	436
11.5	$C^{0,1}$ -Type Estimate	451

Contents	xv
Appendix A: The O_s Notation	455
Appendix B: Function Spaces and Elliptic Equations on Lipschitz Domains	465
Appendix C: The Meyers $L^{2+\delta}$ Estimate	473
Appendix D: Sobolev Norms and Heat Flow	481
Appendix E: Parabolic Green Functions	499
References	511
Index	517