

Contents

Introduction	vii
1 The Laplacian on a Riemannian Manifold	1
1.1 Basic Examples	2
1.1.1 The Laplacian on S^1 and \mathbf{R}	3
1.1.2 Heat Flow on S^1 and \mathbf{R}	5
1.2 The Laplacian on a Riemannian Manifold	10
1.2.1 Riemannian Metrics	10
1.2.2 L^2 Spaces of Functions and Forms	14
1.2.3 The Laplacian on Functions	17
1.3 Hodge Theory for Functions and Forms	22
1.3.1 Analytic Preliminaries	22
1.3.2 The Heat Equation Proof of the Hodge Theorem for Functions	27
1.3.3 The Hodge Theorem for Differential Forms	33
1.3.4 Regularity Results	35
1.4 De Rham Cohomology	39
1.5 The Kernel of the Laplacian on Forms	46
2 Elements of Differential Geometry	52
2.1 Curvature	52
2.2 The Levi-Civita Connection and Bochner Formula	63
2.2.1 The Levi-Civita Connection	63
2.2.2 Weitzenböck Formulas and Gårding's Inequality	67
2.3 Geodesics	79
2.4 The Laplacian in Exponential Coordinates	85
3 The Construction of the Heat Kernel	90
3.1 Preliminary Results for the Heat Kernel	90
3.2 Construction of the Heat Kernel	92
3.2.1 Construction of the Parametrix	92
3.2.2 The Heat Kernel for Functions	96
3.3 The Asymptotics of the Heat Kernel	101

3.4	Positivity of the Heat Kernel	108
4	The Heat Equation Approach to the Atiyah-Singer Index Theorem	111
4.1	The Chern-Gauss-Bonnet Theorem	111
4.1.1	The Heat Equation Approach	112
4.1.2	Proof of the Chern-Gauss-Bonnet Theorem	116
4.2	The Hirzebruch Signature Theorem and the Atiyah-Singer Index Theorem	128
4.2.1	A Survey of Characteristic Forms	128
4.2.2	The Hirzebruch Signature Theorem	134
4.2.3	The Atiyah-Singer Index Theorem	139
5	Zeta Functions of Laplacians	144
5.1	The Zeta Function of a Laplacian	144
5.2	Isospectral Manifolds	151
5.3	Reidemeister Torsion and Analytic Torsion	153
5.3.1	Reidemeister Torsion	153
5.3.2	Analytic Torsion	154
5.3.3	The Families Index Theorem and Analytic Torsion	163
	Bibliography	166
	Index	171