

# Table of Contents

Preface .....	V
Contributors .....	XV

---

## Part I. Visualizing Mathematics

---

### The Minimax Sphere Eversion

George Francis, John M. Sullivan, Rob B. Kusner, Ken A. Brakke, Chris Hartman, and Glenn Chappell .....	3
1. A History of Sphere Eversions .....	3
2. The Minimax Sphere Eversion .....	5
3. Using the Surface Evolver to Evert a Sphere .....	7
4. A Topological Description of the Eversion .....	11
5. Virtual Environments and Supercomputing'95 .....	16

### Exploring Plane Hyperbolic Geometry

Barbara Hausmann, Britta Slopianka, and Hans-Peter Seidel .....	21
1. Introduction .....	21
2. Plane Euclidean and Hyperbolic Geometry .....	22
3. Different Euclidean Models for Hyperbolic Geometry and Geometric Transformations .....	23
4. Hyperbolic Escher Patterns .....	31
5. Software Implementation Details .....	33
6. Conclusion and Further Work .....	35

### Visualizing Nonlinear Electrodynamics

Geoffrey Martin and Ivan Sterling .....	37
1. Introduction .....	37
2. Review of Martin Theory .....	38
3. Balanced Fields for $R^8$ .....	39
4. The First Reduction - to a Euclidean $R^4$ .....	46
5. Electric and Magnetic Fields .....	47
6. Radially Symmetric .....	49

**The Use of Computer Graphics for Solving Problems in Singularity Theory**

Richard J. Morris ..... 53

1. Introduction ..... 53

2. A Toolkit for Exploring Singularity Theory ..... 54

3. Two Problems in Singularity Theory ..... 60

**What Should a Surface in 4-Space Look Like?**

Dennis Roseman ..... 67

1. Introduction ..... 67

2. Mathematical Setting ..... 68

3. What Mathematical Questions Do We Want to Address ? ..... 69

4. How to Improve a Knotted Surface ? ..... 69

5. Visualization in 4-Dimensions: General ..... 70

6. A Concrete Example ..... 72

7. Visualization Solutions in  $\mathbb{R}^4$ : Projections, Slices, and Slabs ..... 72

8. Visualization Solutions in  $\mathbb{R}^4$  : Adding the “Missing” Information . 74

9. Some Current Visualization Tools ..... 78

10. Notes on the Figures ..... 79

**Animation of Algebraic Surfaces**

Dietmar Saupe and Matthias Ruhl ..... 83

1. Introduction ..... 83

2. Raytracing Algebraic Surfaces ..... 85

3. Polygonalization and Physically-based Interactive Animation ..... 88

4. Conclusion ..... 91

---

**Part II. Geometric Algorithms and Experiments**

---

**Using Symmetry Features of the Surface Evolver to Study Foams**

Ken A. Brakke and John M. Sullivan ..... 95

1. Introduction ..... 95

2. The Foams of Kelvin and Weaire–Phelan ..... 96

3. Triply Periodic Foams in the Evolver ..... 98

4. Using Two-fold Symmetries of Minimal Surfaces for the Kelvin Foam ..... 104

5. Other Rotational Symmetries and the Weaire–Phelan Foam ..... 108

6. Using Edge Energies to Eliminate Faces in Mirror Planes ..... 114

## Constant Mean Curvature Surfaces Derived from Delaunay's and Wente's Examples

Karsten Große-Brauckmann and Konrad Polthier	119
1. Introduction	119
2. Delaunay Unduloids and Wente Tori	121
3. Modifying the Wente Torus	122
4. Moduli Spaces of Embedded Symmetric Surfaces with Delaunay Ends	125

## Visualization of Periodic Tilings

Daniel H. Huson	135
1. Tilings and Patterns	135
2. A Fundamental Data-Structure for Periodic Tilings	136
3. Visualization of 2-Dimensional Tilings	137
4. Software	138

## An Algorithm for Discrete Constant Mean Curvature Surfaces

Bernd Oberknapp and Konrad Polthier	141
1. Introduction	141
2. Prerequisites	143
3. Discrete Harmonic Maps and Minimal Surfaces in $S^3$	146
4. Discrete CMC Surfaces via Conjugation	151
5. Conjugate Surface Construction and Symmetry	155
6. Numerical Considerations and Visualization	158

---

## Part III. Visualization Algorithms and Data Structures

---

### Efficient Calculation of Subdivision Surfaces for Visualization

Markus Kohler and Heinrich Müller	165
1. Introduction	165
2. Doo-Sabin Subdivision Surfaces	166
3. Depth-First Subdivision	167
4. Tracing Strategies	176
5. Examples	177

### Fast Line Integral Convolution for Arbitrary Surfaces in 3D

Henrik Battke, Detlev Stalling, and Hans-Christian Hege	181
1. Introduction	181
2. Background	182
3. LIC on Surfaces	184
4. Projecting 3D-Fields	192

5. Applications and Results ..... 193

**Visualization of Parallel Data based on Procedural Access**

Martin Rumpf and Bernhard Schupp ..... 197

1. Introduction ..... 197

2. Review of Domain Decomposition and  
Related Visualization Approaches ..... 198

3. A Procedural Interface to Arbitrary Meshes ..... 199

4. The Parallel Case ..... 201

5. Improving Efficiency by Filtering ..... 204

6. Aspects of Time Scheduling ..... 205

7. Conclusions ..... 206

**Part IV. Visualization Environments**

**A new 3D Graphics Library: Concepts, Implementation, and Examples**

Markus Alefeld, Jörg Haber, and Alexander Heim ..... 211

1. Concepts ..... 211

2. Implementation ..... 216

3. Examples ..... 221

**A Generic Approach to Computer Graphics**

Ekkehard Beier ..... 227

1. Introduction ..... 227

2. Categories of Extensibility ..... 227

3. Generic-3D ..... 229

4. Functionality ..... 232

5. *G* vs. ISO CGRM ..... 233

6. Examples ..... 234

7. How to start with *G* ..... 235

8. Conclusion ..... 236

**MRT – A Visualization Tool Addressing Problems ‘outside’  
the Classical Rendering Domain**

Dieter W. Fellner ..... 239

1. Introduction ..... 239

2. Architecture of MRT ..... 239

3. MRT at Work ..... 244

4. Introducing New Objects ..... 245

5. Experiences .....	246
6. Conclusions .....	248

### **Oorange: A Virtual Laboratory for Experimental Mathematics**

Charles Gunn, Armin Ortmann, Ulrich Pinkall, Konrad Polthier, and Uwe Schwarz .....	249
--	-----

1. Introduction .....	249
2. Overview of Oorange .....	250
3. Components of the Virtual Laboratory .....	252
4. Special Tools .....	261
5. Previous Work .....	262
6. Future Directions .....	263
7. Conclusion .....	263

### **Linear Inductive Reductive Dataflow System for ViSC**

Jacques Lemordant .....	267
-------------------------	-----

1. Introduction .....	267
2. Field Object .....	269
3. Field Scene Object .....	272
4. Inductive Reductive Dataflow .....	274
5. Graphical Scene Object .....	275
6. Example .....	276
7. Conclusions .....	276

### **See what I mean? Using Graphics Toolkits to Visualise Numerical Data**

Jeremy Walton and Michael Dewar .....	279
---------------------------------------	-----

1. Introduction .....	279
2. Graphics Toolkits for Visualisation .....	280
3. A Computer Algebra Package .....	285
4. A Visualisation Web Server .....	290
5. Benchmarking Visualisation Systems .....	290
6. Conclusions .....	296

---

## **Part V. Visualization and Simulation Techniques**

---

### **Numerical Algorithms and Visualization in Medical Treatment Planning**

Rudolf Beck, Peter Deuffhard, Hans-Christian Hege, Martin Seebaß, and Detlev Stalling .....	303
--	-----

1. Introduction .....	303
-----------------------	-----

2. Computer Based Therapy Planning .....	304
3. The Medical Planning System HyperPlan .....	311
4. Conclusions .....	324

**Level Set Methods for Curvature Flow, Image Enhancement, and Shape Recovery in Medical Images**

Ravi Malladi and James A. Sethian .....

329

1. Introduction.....	329
2. Level Set Methods .....	329
3. Motion under Curvature, Self-Similar Flows, and Minimal Surfaces .	334
4. Image Enhancement and Noise Removal .....	338
5. Shape Recovery .....	341

**Numerical Methods, Simulations and Visualization for Compressible Flows**

Monika Wierse, Thomas Geßner, and Dietmar Kröner .....

347

1. Flow Problem in a Simplified Two-Stroke Engine .....	347
2. Numerical Method .....	349
3. Graphical Demands .....	351
4. Further Numerical Test Problems .....	352
5. Analysing the Exchange Process .....	354

**Appendix: Color Plates .....** 357