

Contents

Foreword to the First Edition	v
Foreword to the Second Edition	ix
PART ONE	
Review of Calculus	1
CHAPTER 0	
Sets and Mappings	3
1. Sets	3
2. Mappings	4
3. Natural Numbers and Induction	8
4. Denumerable Sets	11
5. Equivalence Relations	15
CHAPTER I	
Real Numbers	17
1. Algebraic Axioms	17
2. Ordering Axioms	21
3. Integers and Rational Numbers	25
4. The Completeness Axiom	29
CHAPTER II	
Limits and Continuous Functions	34
1. Sequences of Numbers	34
2. Functions and Limits	41
	xi

3. Limits with Infinity	50
4. Continuous Functions	59

CHAPTER III
Differentiation 66

1. Properties of the Derivative	66
2. Mean Value Theorem	70
3. Inverse Functions	74

CHAPTER IV
Elementary Functions 78

1. Exponential	78
2. Logarithm	83
3. Sine and Cosine	90
4. Complex Numbers	95

CHAPTER V
The Elementary Real Integral 101

1. Characterization of the Integral	101
2. Properties of the Integral	104
3. Taylor's Formula	109
4. Asymptotic Estimates and Stirling's Formula	116

PART TWO
Convergence 127

CHAPTER VI
Normed Vector Spaces 129

1. Vector Spaces	129
2. Normed Vector Spaces	131
3. n -Space and Function Spaces	137
4. Completeness	143
5. Open and Closed Sets	151

CHAPTER VII
Limits 160

1. Basic Properties	160
2. Continuous Maps	170
3. Limits in Function Spaces	179
4. Completion of a Normed Vector Space	188

CHAPTER VIII	
Compactness	193
1. Basic Properties of Compact Sets	193
2. Continuous Maps on Compact Sets	197
3. Algebraic Closure of the Complex Numbers	201
4. Relation with Open Coverings	203
CHAPTER IX	
Series	206
1. Basic Definitions	206
2. Series of Positive Numbers	208
3. Non-Absolute Convergence	217
4. Absolute Convergence in Vector Spaces	225
5. Absolute and Uniform Convergence	229
6. Power Series	234
7. Differentiation and Integration of Series	239
CHAPTER X	
The Integral in One Variable	246
1. Extension Theorem for Linear Maps	246
2. Integral of Step Maps	248
3. Approximation by Step Maps	252
4. Properties of the Integral	255
Appendix. The Lebesgue Integral	262
5. The Derivative	267
6. Relation Between the Integral and the Derivative	272
7. Interchanging Derivatives and Integrals	275
PART THREE	
Applications of the Integral	281
CHAPTER XI	
Approximation with Convolutions	283
1. Dirac Sequences	283
2. The Weierstrass Theorem	287
CHAPTER XII	
Fourier Series	291
1. Hermitian Products and Orthogonality	291
2. Trigonometric Polynomials as a Total Family	306
3. Explicit Uniform Approximation	311
4. Pointwise Convergence	317

CHAPTER XIII	
Improper Integrals	326
1. Definition	326
2. Criteria for Convergence	330
3. Interchanging Derivatives and Integrals	336
4. The Heat Kernel	347
CHAPTER XIV	
The Fourier Integral	353
1. The Schwartz Space	353
2. The Fourier Inversion Formula	359
3. An Example of Fourier Transform not in the Schwartz Space	363
PART FOUR	
Calculus in Vector Spaces	369
CHAPTER XV	
Functions on n-Space	371
1. Partial Derivatives	371
2. Differentiability and the Chain Rule	379
3. Potential Functions	388
4. Curve Integrals	395
5. Taylor's Formula	405
6. Maxima and the Derivative	411
CHAPTER XVI	
The Winding Number and Global Potential Functions	417
1. Another Description of the Integral Along a Path	418
2. The Winding Number and Homology	420
3. Proof of the Global Integrability Theorem	432
4. The Integral Over Continuous Paths	438
5. The Homotopy Form of the Integrability Theorem	444
6. More on Homotopies	450
CHAPTER XVII	
Derivatives in Vector Spaces	455
1. The Space of Continuous Linear Maps	455
2. The Derivative as a Linear Map	463
3. Properties of the Derivative	468
4. Mean Value Theorem	473

5. The Second Derivative	477
6. Higher Derivatives and Taylor's Formula	487
7. Partial Derivatives	495
8. Differentiating Under the Integral Sign	499

CHAPTER XVIII

Inverse Mapping Theorem	502
1. The Shrinking Lemma	502
2. Inverse Mappings, Linear Case	506
3. The Inverse Mapping Theorem	512
4. Implicit Functions and Charts	520
5. Product Decompositions	526

CHAPTER XIX

Ordinary Differential Equations	538
1. Local Existence and Uniqueness	538
2. Approximate Solutions	548
3. Linear Differential Equations	552
4. Dependence on Initial Conditions	557

PART FIVE

Multiple Integration	563
---------------------------------------	-----

CHAPTER XX

Multiple Integrals	565
1. Elementary Multiple Integration	565
2. Criteria for Admissibility	578
3. Repeated Integrals	581
4. Change of Variables	584
5. Vector Fields on Spheres	602

CHAPTER XXI

Differential Forms	607
1. Definitions	607
2. Stokes' Theorem for a Rectangle	613
3. Inverse Image of a Form	616
4. Stokes' Formula for Simplices	620

Appendix	627
---------------------------	-----

Index	635
------------------------	-----