

Contents

Preface

xvii

1 Preliminary Information

1

- 1.1 Mathematical Preliminaries, 1
 - 1.1.1 Factorial and Combinatorial Conventions, 1
 - 1.1.2 Gamma and Beta Functions, 5
 - 1.1.3 Finite Difference Calculus, 10
 - 1.1.4 Differential Calculus, 14
 - 1.1.5 Incomplete Gamma and Beta Functions and Other Gamma-Related Functions, 16
 - 1.1.6 Gaussian Hypergeometric Functions, 20
 - 1.1.7 Confluent Hypergeometric Functions (Kummer's Functions), 23
 - 1.1.8 Generalized Hypergeometric Functions, 26
 - 1.1.9 Bernoulli and Euler Numbers and Polynomials, 29
 - 1.1.10 Integral Transforms, 32
 - 1.1.11 Orthogonal Polynomials, 32
 - 1.1.12 Basic Hypergeometric Series, 34
- 1.2 Probability and Statistical Preliminaries, 37
 - 1.2.1 Calculus of Probabilities, 37
 - 1.2.2 Bayes's Theorem, 41
 - 1.2.3 Random Variables, 43
 - 1.2.4 Survival Concepts, 45
 - 1.2.5 Expected Values, 47
 - 1.2.6 Inequalities, 49
 - 1.2.7 Moments and Moment Generating Functions, 50
 - 1.2.8 Cumulants and Cumulant Generating Functions, 54

- 1.2.9 Joint Moments and Cumulants, 56
- 1.2.10 Characteristic Functions, 57
- 1.2.11 Probability Generating Functions, 58
- 1.2.12 Order Statistics, 61
- 1.2.13 Truncation and Censoring, 62
- 1.2.14 Mixture Distributions, 64
- 1.2.15 Variance of a Function, 65
- 1.2.16 Estimation, 66
- 1.2.17 General Comments on the Computer Generation of Discrete Random Variables, 71
- 1.2.18 Computer Software, 73

2 Families of Discrete Distributions

74

- 2.1 Lattice Distributions, 74
- 2.2 Power Series Distributions, 75
 - 2.2.1 Generalized Power Series Distributions, 75
 - 2.2.2 Modified Power Series Distributions, 79
- 2.3 Difference-Equation Systems, 82
 - 2.3.1 Katz and Extended Katz Families, 82
 - 2.3.2 Sundt and Jewell Family, 85
 - 2.3.3 Ord's Family, 87
- 2.4 Kemp Families, 89
 - 2.4.1 Generalized Hypergeometric Probability Distributions, 89
 - 2.4.2 Generalized Hypergeometric Factorial Moment Distributions, 96
- 2.5 Distributions Based on Lagrangian Expansions, 99
- 2.6 Gould and Abel Distributions, 101
- 2.7 Factorial Series Distributions, 103
- 2.8 Distributions of Order- k , 105
- 2.9 q -Series Distributions, 106

3 Binomial Distribution

108

- 3.1 Definition, 108
- 3.2 Historical Remarks and Genesis, 109
- 3.3 Moments, 109
- 3.4 Properties, 112
- 3.5 Order Statistics, 116
- 3.6 Approximations, Bounds, and Transformations, 116

- 3.6.1 Approximations, 116
- 3.6.2 Bounds, 122
- 3.6.3 Transformations, 123
- 3.7 Computation, Tables, and Computer Generation, 124
 - 3.7.1 Computation and Tables, 124
 - 3.7.2 Computer Generation, 125
- 3.8 Estimation, 126
 - 3.8.1 Model Selection, 126
 - 3.8.2 Point Estimation, 126
 - 3.8.3 Confidence Intervals, 130
 - 3.8.4 Model Verification, 133
- 3.9 Characterizations, 134
- 3.10 Applications, 135
- 3.11 Truncated Binomial Distributions, 137
- 3.12 Other Related Distributions, 140
 - 3.12.1 Limiting Forms, 140
 - 3.12.2 Sums and Differences of Binomial-Type Variables, 140
 - 3.12.3 Poissonian Binomial, Lexian, and Coolidge Schemes, 144
 - 3.12.4 Weighted Binomial Distributions, 149
 - 3.12.5 Chain Binomial Models, 151
 - 3.12.6 Correlated Binomial Variables, 151

4 Poisson Distribution

156

- 4.1 Definition, 156
- 4.2 Historical Remarks and Genesis, 156
 - 4.2.1 Genesis, 156
 - 4.2.2 Poissonian Approximations, 160
- 4.3 Moments, 161
- 4.4 Properties, 163
- 4.5 Approximations, Bounds, and Transformations, 167
- 4.6 Computation, Tables, and Computer Generation, 170
 - 4.6.1 Computation and Tables, 170
 - 4.6.2 Computer Generation, 171
- 4.7 Estimation, 173
 - 4.7.1 Model Selection, 173
 - 4.7.2 Point Estimation, 174
 - 4.7.3 Confidence Intervals, 176

- 4.7.4 Model Verification, 178
- 4.8 Characterizations, 179
- 4.9 Applications, 186
- 4.10 Truncated and Misrecorded Poisson Distributions, 188
 - 4.10.1 Left Truncation, 188
 - 4.10.2 Right Truncation and Double Truncation, 191
 - 4.10.3 Misrecorded Poisson Distributions, 193
- 4.11 Poisson–Stopped Sum Distributions, 195
- 4.12 Other Related Distributions, 196
 - 4.12.1 Normal Distribution, 196
 - 4.12.2 Gamma Distribution, 196
 - 4.12.3 Sums and Differences of Poisson Variates, 197
 - 4.12.4 Hyper-Poisson Distributions, 199
 - 4.12.5 Grouped Poisson Distributions, 202
 - 4.12.6 Heine and Euler Distributions, 205
 - 4.12.7 Intervened Poisson Distributions, 205

5 Negative Binomial Distribution

208

- 5.1 Definition, 208
- 5.2 Geometric Distribution, 210
- 5.3 Historical Remarks and Genesis of Negative Binomial Distribution, 212
- 5.4 Moments, 215
- 5.5 Properties, 217
- 5.6 Approximations and Transformations, 218
- 5.7 Computation and Tables, 220
- 5.8 Estimation, 222
 - 5.8.1 Model Selection, 222
 - 5.8.2 P Unknown, 222
 - 5.8.3 Both Parameters Unknown, 223
 - 5.8.4 Data Sets with a Common Parameter, 226
 - 5.8.5 Recent Developments, 227
- 5.9 Characterizations, 228
 - 5.9.1 Geometric Distribution, 228
 - 5.9.2 Negative Binomial Distribution, 231
- 5.10 Applications, 232
- 5.11 Truncated Negative Binomial Distributions, 233
- 5.12 Related Distributions, 236
 - 5.12.1 Limiting Forms, 236

- 5.12.2 Extended Negative Binomial Model, 237
- 5.12.3 Lagrangian Generalized Negative Binomial Distribution, 239
- 5.12.4 Weighted Negative Binomial Distributions, 240
- 5.12.5 Convolutions Involving Negative Binomial Variates, 241
- 5.12.6 Pascal–Poisson Distribution, 243
- 5.12.7 Minimum (Riff–Shuffle) and Maximum Negative Binomial Distributions, 244
- 5.12.8 Condensed Negative Binomial Distributions, 246
- 5.12.9 Other Related Distributions, 247

6 Hypergeometric Distributions

251

- 6.1 Definition, 251
- 6.2 Historical Remarks and Genesis, 252
 - 6.2.1 Classical Hypergeometric Distribution, 252
 - 6.2.2 Beta–Binomial Distribution, Negative (Inverse) Hypergeometric Distribution: Hypergeometric Waiting-Time Distribution, 253
 - 6.2.3 Beta–Negative Binomial Distribution: Beta–Pascal Distribution, Generalized Waring Distribution, 256
 - 6.2.4 Pólya Distributions, 258
 - 6.2.5 Hypergeometric Distributions in General, 259
- 6.3 Moments, 262
- 6.4 Properties, 265
- 6.5 Approximations and Bounds, 268
- 6.6 Tables, Computation, and Computer Generation, 271
- 6.7 Estimation, 272
 - 6.7.1 Classical Hypergeometric Distribution, 273
 - 6.7.2 Negative (Inverse) Hypergeometric Distribution: Beta–Binomial Distribution, 274
 - 6.7.3 Beta–Pascal Distribution, 276
- 6.8 Characterizations, 277
- 6.9 Applications, 279
 - 6.9.1 Classical Hypergeometric Distribution, 279
 - 6.9.2 Negative (Inverse) Hypergeometric Distribution: Beta–Binomial Distribution, 281
 - 6.9.3 Beta–Negative Binomial Distribution: Beta–Pascal Distribution, Generalized Waring Distribution, 283
- 6.10 Special Cases, 283

- 6.10.1 Discrete Rectangular Distribution, 283
- 6.10.2 Distribution of Leads in Coin Tossing, 286
- 6.10.3 Yule Distribution, 287
- 6.10.4 Waring Distribution, 289
- 6.10.5 Narayana Distribution, 291
- 6.11 Related Distributions, 293
 - 6.11.1 Extended Hypergeometric Distributions, 293
 - 6.11.2 Generalized Hypergeometric Probability Distributions, 296
 - 6.11.3 Generalized Hypergeometric Factorial Moment Distributions, 298
 - 6.11.4 Other Related Distributions, 299

7 Logarithmic and Lagrangian Distributions

302

- 7.1 Logarithmic Distribution, 302
 - 7.1.1 Definition, 302
 - 7.1.2 Historical Remarks and Genesis, 303
 - 7.1.3 Moments, 305
 - 7.1.4 Properties, 307
 - 7.1.5 Approximations and Bounds, 309
 - 7.1.6 Computation, Tables, and Computer Generation, 310
 - 7.1.7 Estimation, 311
 - 7.1.8 Characterizations, 315
 - 7.1.9 Applications, 316
 - 7.1.10 Truncated and Modified Logarithmic Distributions, 317
 - 7.1.11 Generalizations of the Logarithmic Distribution, 319
 - 7.1.12 Other Related Distributions, 321
- 7.2 Lagrangian Distributions, 325
 - 7.2.1 Otter's Multiplicative Process, 326
 - 7.2.2 Borel Distribution, 328
 - 7.2.3 Consul Distribution, 329
 - 7.2.4 Geeta Distribution, 330
 - 7.2.5 General Lagrangian Distributions of the First Kind, 331
 - 7.2.6 Lagrangian Poisson Distribution, 336
 - 7.2.7 Lagrangian Negative Binomial Distribution, 340

- 7.2.8 Lagrangian Logarithmic Distribution, 341
- 7.2.9 Lagrangian Distributions of the Second Kind, 342

8 Mixture Distributions

343

- 8.1 Basic Ideas, 343
 - 8.1.1 Introduction, 343
 - 8.1.2 Finite Mixtures, 344
 - 8.1.3 Varying Parameters, 345
 - 8.1.4 Bayesian Interpretation, 347
- 8.2 Finite Mixtures of Discrete Distributions, 347
 - 8.2.1 Parameters of Finite Mixtures, 347
 - 8.2.2 Parameter Estimation, 349
 - 8.2.3 Zero-Modified and Hurdle Distributions, 351
 - 8.2.4 Examples of Zero-Modified Distributions, 353
 - 8.2.5 Finite Poisson Mixtures, 357
 - 8.2.6 Finite Binomial Mixtures, 358
 - 8.2.7 Other Finite Mixtures of Discrete Distributions, 359
- 8.3 Continuous and Countable Mixtures of Discrete Distributions, 360
 - 8.3.1 Properties of General Mixed Distributions, 360
 - 8.3.2 Properties of Mixed Poisson Distributions, 362
 - 8.3.3 Examples of Poisson Mixtures, 365
 - 8.3.4 Mixtures of Binomial Distributions, 373
 - 8.3.5 Examples of Binomial Mixtures, 374
 - 8.3.6 Other Continuous and Countable Mixtures of Discrete Distributions, 376
- 8.4 Gamma and Beta Mixing Distributions, 378

9 Stopped-Sum Distributions

381

- 9.1 Generalized and Generalizing Distributions, 381
- 9.2 Damage Processes, 386
- 9.3 Poisson–Stopped Sum (Multiple Poisson) Distributions, 388
- 9.4 Hermite Distribution, 394
- 9.5 Poisson–Binomial Distribution, 400
- 9.6 Neyman Type A Distribution, 403
 - 9.6.1 Definition, 403
 - 9.6.2 Moment Properties, 405
 - 9.6.3 Tables and Approximations, 406

- 9.6.4 Estimation, 407
- 9.6.5 Applications, 409
- 9.7 Pólya–Aeppli Distribution, 410
- 9.8 Generalized Pólya–Aeppli (Poisson–Negative Binomial) Distribution, 414
- 9.9 Generalizations of Neyman Type A Distribution, 416
- 9.10 Thomas Distribution, 421
- 9.11 Borel–Tanner Distribution: Lagrangian Poisson Distribution, 423
- 9.12 Other Poisson–Stopped Sum (multiple Poisson) Distributions, 425
- 9.13 Other Families of Stopped-Sum Distributions, 426

10 Matching, Occupancy, Runs, and q -Series Distributions

430

- 10.1 Introduction, 430
- 10.2 Probabilities of Combined Events, 431
- 10.3 Matching Distributions, 434
- 10.4 Occupancy Distributions, 439
 - 10.4.1 Classical Occupancy and Coupon Collecting, 439
 - 10.4.2 Maxwell–Boltzmann, Bose–Einstein, and Fermi–Dirac Statistics, 444
 - 10.4.3 Specified Occupancy and Grassia–Binomial Distributions, 446
- 10.5 Record Value Distributions, 448
- 10.6 Runs Distributions, 450
 - 10.6.1 Runs of Like Elements, 450
 - 10.6.2 Runs Up and Down, 453
- 10.7 Distributions of Order k , 454
 - 10.7.1 Early Work on Success Runs Distributions, 454
 - 10.7.2 Geometric Distribution of Order k , 456
 - 10.7.3 Negative Binomial Distributions of Order k , 458
 - 10.7.4 Poisson and Logarithmic Distributions of Order k , 459
 - 10.7.5 Binomial Distributions of Order k , 461
 - 10.7.6 Further Distributions of Order k , 463
- 10.8 q -Series Distributions, 464
 - 10.8.1 Terminating Distributions, 465
 - 10.8.2 q -Series Distributions with Infinite Support, 470
 - 10.8.3 Bilateral q -Series Distributions, 474
 - 10.8.4 q -Series Related Distributions, 476

- 11.1 Parametric Regression Models, 478
 - 11.1.1 Introduction, 478
 - 11.1.2 Tweedie–Poisson Family, 480
 - 11.1.3 Negative Binomial Regression Models, 482
 - 11.1.4 Poisson Lognormal Model, 483
 - 11.1.5 Poisson–Inverse Gaussian (Sichel) Model, 484
 - 11.1.6 Poisson Polynomial Distribution, 487
 - 11.1.7 Weighted Poisson Distributions, 488
 - 11.1.8 Double-Poisson and Double-Binomial Distributions, 489
 - 11.1.9 Simplex–Binomial Mixture Model, 490
- 11.2 Miscellaneous Discrete Distributions, 491
 - 11.2.1 Dandekar’s Modified Binomial and Poisson Models, 491
 - 11.2.2 Digamma and Trigamma Distributions, 492
 - 11.2.3 Discrete Adès Distribution, 494
 - 11.2.4 Discrete Bessel Distribution, 495
 - 11.2.5 Discrete Mittag–Leffler Distribution, 496
 - 11.2.6 Discrete Student’s t Distribution, 498
 - 11.2.7 Feller–Arley and Gegenbauer Distributions, 499
 - 11.2.8 Gram–Charlier Type B Distributions, 501
 - 11.2.9 “Interrupted” Distributions, 502
 - 11.2.10 Lost-Games Distributions, 503
 - 11.2.11 Luria–Delbrück Distribution, 505
 - 11.2.12 Naor’s Distribution, 507
 - 11.2.13 Partial-Sums Distributions, 508
 - 11.2.14 Queueing Theory Distributions, 512
 - 11.2.15 Reliability and Survival Distributions, 514
 - 11.2.16 Skellam–Haldane Gene Frequency Distribution, 519
 - 11.2.17 Steyn’s Two-Parameter Power Series Distributions, 521
 - 11.2.18 Univariate Multinomial-Type Distributions, 522
 - 11.2.19 Urn Models with Stochastic Replacements, 524
 - 11.2.20 Zipf-Related Distributions, 526
 - 11.2.21 Haight’s Zeta Distributions, 533

Bibliography 535

Abbreviations 631

Index 633