

# Contents

Preface to the Fourth Edition	vii
From Preface to the Third Edition	ix
To the Reader	xv
CHAPTER 1	
Groups and Homomorphisms	1
Permutations	2
Cycles	3
Factorization into Disjoint Cycles	6
Even and Odd Permutations	7
Semigroups	10
Groups	12
Homomorphisms	16
CHAPTER 2	
The Isomorphism Theorems	20
Subgroups	20
Lagrange's Theorem	24
Cyclic Groups	28
Normal Subgroups	29
Quotient Groups	32
The Isomorphism Theorems	35
Correspondence Theorem	37
Direct Products	40

CHAPTER 3	
Symmetric Groups and $G$ -Sets	43
Conjugates	43
Symmetric Groups	46
The Simplicity of $A_n$	50
Some Representation Theorems	51
$G$ -Sets	55
Counting Orbits	58
Some Geometry	63
CHAPTER 4	
The Sylow Theorems	73
$p$ -Groups	73
The Sylow Theorems	78
Groups of Small Order	82
CHAPTER 5	
Normal Series	89
Some Galois Theory	91
The Jordan–Hölder Theorem	98
Solvable Groups	102
Two Theorems of P. Hall	108
Central Series and Nilpotent Groups	112
$p$ -Groups	119
CHAPTER 6	
Finite Direct Products	125
The Basis Theorem	125
The Fundamental Theorem of Finite Abelian Groups	131
Canonical Forms; Existence	133
Canonical Forms; Uniqueness	141
The Krull–Schmidt Theorem	144
Operator Groups	151
CHAPTER 7	
Extensions and Cohomology	154
The Extension Problem	154
Automorphism Groups	156
Semidirect Products	167
Wreath Products	172
Factor Sets	178
Theorems of Schur–Zassenhaus and Gaschütz	188
Transfer and Burnside’s Theorem	193
Projective Representations and the Schur Multiplier	201
Derivations	211

Contents	xiii
<b>CHAPTER 8</b>	
<b>Some Simple Linear Groups</b>	217
Finite Fields	217
The General Linear Group	219
$\text{PSL}(2, K)$	224
$\text{PSL}(m, K)$	227
Classical Groups	234
<b>CHAPTER 9</b>	
<b>Permutations and the Mathieu Groups</b>	247
Multiple Transitivity	247
Primitive $G$ -Sets	256
Simplicity Criteria	259
Affine Geometry	264
Projective Geometry	272
Sharply 3-Transitive Groups	281
Mathieu Groups	286
Steiner Systems	293
<b>CHAPTER 10</b>	
<b>Abelian Groups</b>	307
Basics	307
Free Abelian Groups	312
Finitely Generated Abelian Groups	318
Divisible and Reduced Groups	320
Torsion Groups	325
Subgroups of $\mathbb{Q}$	331
Character Groups	335
<b>CHAPTER 11</b>	
<b>Free Groups and Free Products</b>	343
Generators and Relations	343
Semigroup Interlude	349
Coset Enumeration	351
Presentations and the Schur Multiplier	358
Fundamental Groups of Complexes	366
Tietze's Theorem	374
Covering Complexes	377
The Nielsen-Schreier Theorem	383
Free Products	388
The Kurosh Theorem	391
The van Kampen Theorem	394
Amalgams	401
HNN Extensions	407

CHAPTER 12	
The Word Problem	418
Introduction	418
Turing Machines	420
The Markov–Post Theorem	425
The Novikov–Boone–Britton Theorem: Sufficiency of Boone’s Lemma	430
Cancellation Diagrams	433
The Novikov–Boone–Britton Theorem: Necessity of Boone’s Lemma	438
The Higman Imbedding Theorem	450
Some Applications	464
Epilogue	471
APPENDIX I	
Some Major Algebraic Systems	475
APPENDIX II	
Equivalence Relations and Equivalence Classes	477
APPENDIX III	
Functions	479
APPENDIX IV	
Zorn’s Lemma	481
APPENDIX V	
Countability	483
APPENDIX VI	
Commutative Rings	485
Bibliography	495
Notation	498
Index	503