CONTENTS

Preface xvii

1 A BRIEF INTRODUCTION TO INFORMATION THEORY 1

Classical Information 1
Information Content in a Signal 2
Entropy and Shannon's Information Theory 3
Probability Basics 7
Example 1.1 8
Solution 8
Exercises 8

2 QUBITS AND QUANTUM STATES 11

The Qubit 11
Example 2.1 13
Solution 13
Vector Spaces 14
Example 2.2 16
Solution 17
Linear Combinations of Vectors 17
Example 2.3 18
Solution 18
Uniqueness of a Spanning Set 19
Basis and Dimension 20
Inner Products 21
Example 2.4 22
Solution 23
Example 2.5 24
Solution 24
Orthonormality 24
Gram-Schmidt Orthogonalization 26
Example 2.6 26
Solution 26

Bra-Ket Formalism 28
Example 2.7 29
Solution 29
The Cauchy-Schwartz and Triangle Inequalities 31
Example 2.8 32
Solution 32
Example 2.9 33
Solution 34
Summary 35
Exercises 36

3 MATRICES AND OPERATORS 39

Observables 40
The Pauli Operators 40
Outer Products 41
Example 3.1 41
Solution 41
You Try It 42
The Closure Relation 42
Representations of Operators Using Matrices 42
Outer Products and Matrix Representations 43
You Try It 44
Matrix Representation of Operators in Two-Dimensional Spaces 44
Example 3.2 44
Solution 44
You Try It 45
Definition: The Pauli Matrices 45
Example 3.3 45
Solution 45
Hermitian, Unitary, and Normal Operators 46
Example 3.4 47
Solution 47
You Try It 47
Definition: Hermitian Operator 47
Definition: Unitary Operator 48
Definition: Normal Operator 48
Eigenvalues and Eigenvectors 48
The Characteristic Equation 49
Example 3.5 49
Solution 49
You Try It 50
Example 3.6 50
Solution 50
Spectral Decomposition 53
Example 3.7 53

Solution 54
The Trace of an Operator 54
Example 3.8 54
Solution 54
Example 3.9 55
Solution 55
Important Properties of the Trace 56
Example 3.10 56
Solution 56
Example 3.11 57
Solution 57
The Expectation Value of an Operator 57
Example 3.12 57
Solution 58
Example 3.13 58
Solution 59
Functions of Operators 59
Unitary Transformations 60
Example 3.14 61
Solution 61
Projection Operators 62
Example 3.15 63
Solution 63
You Try It 63
Example 3.16 65
Solution 65
Positive Operators 66
Commutator Algebra 66
Example 3.17 67
Solution 67
The Heisenberg Uncertainty Principle 68
Polar Decomposition and Singular Values 69
Example 3.18 69
Solution 70
The Postulates of Quantum Mechanics 70
Postulate 1: The State of a System 70
Postulate 2: Observable Quantities Represented by Operators 70
Postulate 3: Measurements 70
Postulate 4: Time Evolution of the System 71
Exercises 71

4 TENSOR PRODUCTS 73

Representing Composite States in Quantum Mechanics 74
Example 4.1 74
Solution 74

Example 4.2 75
Solution 75
Computing Inner Products 76
Example 4.3 76
Solution 76
You Try It 76
Example 4.4 77
Solution 77
You Try It 77
Example 4.5 77
Solution 77
You Try It 77
Tensor Products of Column Vectors 78
Example 4.6 78
Solution 78
You Try It 78
Operators and Tensor Products 79
Example 4.7 79
Solution 79
You Try It 79
Example 4.8 80
Solution 80
Example 4.9 80
Solution 81
Example 4.10 81
Solution 81
You Try It 82
Example 4.11 82
Solution 82
You Try It 82
Tensor Products of Matrices 83
Example 4.12 83
Solution 83
You Try It 84
Exercises 84

5 THE DENSITY OPERATOR 85

The Density Operator for a Pure State 86
Definition: Density Operator for a Pure State 87
Definition: Using the Density Operator to Find the Expectation Value 88
Example 5.1 88
Solution 89
You Try It 89
Time Evolution of the Density Operator 90
Definition: Time Evolution of the Density Operator 91

The Density Operator for a Mixed State 91
Key Properties of a Density Operator 92
Example 5.2 93
Solution 93
Expectation Values 95
Probability of Obtaining a Given Measurement Result 95
Example 5.3 96
Solution 96
You Try It 96
Example 5.4 96
Solution 97
You Try It 98
You Try It 99
You Try It 99
Characterizing Mixed States 99
Example 5.5 100
Solution 100
Example 5.6 102
Solution 103
You Try It 103
Example 5.7 103
Solution 104
Example 5.8 105
Solution 105
Example 5.9 106
Solution 106
You Try It 108
Probability of Finding an Element of the Ensemble in a Given State 108
Example 5.10 109
Solution 109
Completely Mixed States 111
The Partial Trace and the Reduced Density Operator 111
You Try It 113
Example 5.11 114
Solution 114
The Density Operator and the Bloch Vector 115
Example 5.12 116
Solution 116
Exercises 117

6 QUANTUM MEASUREMENT THEORY 121

Distinguishing Quantum States and Measurement 121
Projective Measurements 123
Example 6.1 125
Solution 126

Example 6.2 128
Solution 129
You Try It 130
Example 6.3 130
Solution 130
Measurements on Composite Systems 132
Example 6.4 132
Solution 132
Example 6.5 133
Solution 134
Example 6.6 135
Solution 135
You Try It 136
Example 6.7 136
Solution 137
You Try It 138
Example 6.8 138
Solution 138
Generalized Measurements 139
Example 6.9 140
Solution 140
Example 6.10 140
Solution 140
Positive Operator-Valued Measures 141
Example 6.11 141
Solution 142
Example 6.12 142
Solution 143
Example 6.13 143
Solution 144
Exercises 145

7 ENTANGLEMENT 147

Bell's Theorem 151
Bipartite Systems and the Bell Basis 155
Example 7.1 157
Solution 157
When Is a State Entangled? 157
Example 7.2 158
Solution 158
Example 7.3 158
Solution 158
Example 7.4 159
Solution 159
You Try It 162

You Try It 162
The Pauli Representation 162
Example 7.5 162
Solution 162
Example 7.6 163
Solution 163
Entanglement Fidelity 166
Using Bell States For Density Operator Representation 166
Example 7.7 167
Solution 167
Schmidt Decomposition 168
Example 7.8 168
Solution 168
Example 7.9 169
Solution 169
Purification 169
Exercises 170

8 QUANTUM GATES AND CIRCUITS 173

Classical Logic Gates 173
You Try It 175
Single-Qubit Gates 176
Example 8.1 178
Solution 178
You Try It 179
Example 8.2 179
Solution 180
More Single-Qubit Gates 180
You Try It 181
Example 8.3 181
Solution 181
Example 8.4 182
Solution 182
You Try It 183
Exponentiation 183
Example 8.5 183
Solution 183
You Try It 184
The Z–Y Decomposition 185
Basic Quantum Circuit Diagrams 185
Controlled Gates 186
Example 8.6 187
Solution 188
Example 8.7 188
Solution 188

Example 8.8 190
Solution 190
Example 8.9 191
Solution 192
Gate Decomposition 192
Exercises 195

9 QUANTUM ALGORITHMS 197

Hadamard Gates 198
Example 9.1 200
Solution 201
The Phase Gate 201
Matrix Representation of Serial and Parallel Operations 201
Quantum Interference 202
Quantum Parallelism and Function Evaluation 203
Deutsch-Jozsa Algorithm 207
Example 9.2 208
Solution 208
Example 9.3 209
Solution 209
Quantum Fourier Transform 211
Phase Estimation 213
Shor's Algorithm 216
Quantum Searching and Grover's Algorithm 218
Exercises 221

10 APPLICATIONS OF ENTANGLEMENT: TELEPORTATION AND SUPERDENSE CODING 225

Teleportation 226
Teleportation Step 1: Alice and Bob Share an Entangled Pair of Particles 226
Teleportation Step 2: Alice Applies a CNOT Gate 226
Teleportation Step 3: Alice Applies a Hadamard Gate 227
Teleportation Step 4: Alice Measures Her Pair 227
Teleportation Step 5: Alice Contacts Bob on a Classical Communications Channel and Tells Him Her Measurement Result 228
The Peres Partial Transposition Condition 229
Example 10.1 229
Solution 230
Example 10.2 230
Solution 231
Example 10.3 232
Solution 232
Entanglement Swapping 234
Superdense Coding 236

Example 10.4 237
Solution 237
Exercises 238

11 QUANTUM CRYPTOGRAPHY 239

A Brief Overview of RSA Encryption 241
Example 11.1 242
Solution 242
Basic Quantum Cryptography 243
Example 11.2 245
Solution 245
An Example Attack: The Controlled NOT Attack 246
The B92 Protocol 247
The E91 Protocol (Ekert) 248
Exercises 249

12 QUANTUM NOISE AND ERROR CORRECTION 251

Single-Qubit Errors 252
Quantum Operations and Krauss Operators 254
Example 12.1 255
Solution 255
Example 12.2 257
Solution 257
Example 12.3 259
Solution 259
The Depolarization Channel 260
The Bit Flip and Phase Flip Channels 261
Amplitude Damping 262
Example 12.4 265
Solution 265
Phase Damping 270
Example 12.5 271
Solution 271
Quantum Error Correction 272
Exercises 277

13 TOOLS OF QUANTUM INFORMATION THEORY 279

The No-Cloning Theorem 279
Trace Distance 281
Example 13.1 282
Solution 282
You Try It 283
Example 13.2 283

Solution 284
Example 13.3 285
Solution 285
Fidelity 286
Example 13.4 287
Solution 288
Example 13.5 289
Solution 289
Example 13.6 289
Solution 289
Example 13.7 290
Solution 290
Entanglement of Formation and Concurrence 291
Example 13.8 291
Solution 292
Example 13.9 293
Solution 293
Example 13.10 294
Solution 294
Example 13.11 295
Solution 295
You Try It 296
Information Content and Entropy 296
Example 13.12 298
Solution 298
Example 13.13 299
Solution 299
Example 13.14 299
Solution 299
Example 13.15 300
Solution 300
Example 13.16 301
Solution 301
Example 13.17 302
Solution 302
Exercises 303

14 ADIABATIC QUANTUM COMPUTATION 305

Example 14.1 307
Solution 307
Adiabatic Processes 308
Example 14.2 308
Solution 309
Adiabatic Quantum Computing 310
Example 14.3 310

Solution 310
Exercises 313

15 CLUSTER STATE QUANTUM COMPUTING 315

Cluster States 316
Cluster State Preparation 316
Example 15.1 317
Solution 317
Adjacency Matrices 319
Stabilizer States 320
Aside: Entanglement Witness 322
Cluster State Processing 324
Example 15.2 326
Exercises 326

References 329

Index 331