

CONTENTS

Part 1 The classical picture of turbulence

	7
1. The ubiquitous nature of turbulence	3
1.1 The experiments of Taylor and Bénard	4
1.2 Flow over a cylinder	7
1.3 Reynolds' experiment	8
1.4 Common themes	9
1.5 The ubiquitous nature of turbulence	13
1.6 Different scales in a turbulent flow: a glimpse at the energy cascade of Kolmogorov and Richardson	18
1.7 The closure problem of turbulence	27
1.8 Is there a 'theory of turbulence'?	29
1.9 The interaction of theory, computation, and experiment	30
2. The equations of fluid mechanics	34
2.1 The Navier–Stokes equation	35
2.1.1 <i>Newton's second law applied to a fluid</i>	35
2.1.2 <i>The convective derivative</i>	38
2.1.3 <i>Integral versions of the momentum equation</i>	40
2.1.4 <i>The rate of dissipation of energy in a viscous fluid</i>	41
2.2 Relating pressure to velocity	43
2.3 Vorticity dynamics	44
2.3.1 <i>Vorticity and angular momentum</i>	44
2.3.2 <i>The vorticity equation</i>	48
2.3.3 <i>Kelvin's theorem</i>	52
2.3.4 <i>Tracking vorticity distributions</i>	55
2.4 A definition of turbulence	57
3. The origins and nature of turbulence	61
3.1 The nature of chaos	62
3.1.1 <i>From non-linearity to chaos</i>	63
3.1.2 <i>More on bifurcations</i>	66
3.1.3 <i>The arrow of time</i>	69

3.2	Some elementary properties of freely evolving turbulence	71
3.2.1	<i>Various stages of development</i>	73
3.2.2	<i>The rate of destruction of energy in fully developed turbulence</i>	77
3.2.3	<i>How much does the turbulence remember?</i>	81
3.2.4	<i>The need for a statistical approach and different methods of taking averages</i>	85
3.2.5	<i>Velocity correlations, structure functions, and the energy spectrum</i>	88
3.2.6	<i>Is the asymptotic state universal? Kolmogorov's theory</i>	94
3.2.7	<i>The probability distribution of the velocity field</i>	97
4.	Turbulent shear flows and simple closure models	105
4.1	The exchange of energy between the mean flow and the turbulence	107
4.1.1	<i>Reynolds stresses and the closure problem of turbulence</i>	108
4.1.2	<i>The eddy viscosity theories of Boussinesq and Prandtl</i>	111
4.1.3	<i>The transfer of energy from the mean flow to the turbulence</i>	114
4.1.4	<i>A glimpse at the k-ϵ model</i>	119
4.2	Wall-bounded shear flows and the log-law of the wall	122
4.2.1	<i>Turbulent flow in a channel and the log-law of the wall</i>	122
4.2.2	<i>Inactive motion—a problem for the log-law?</i>	127
4.2.3	<i>Turbulence profiles in channel flow</i>	129
4.2.4	<i>The log-law for a rough wall</i>	131
4.2.5	<i>The structure of a turbulent boundary layer</i>	131
4.2.6	<i>Coherent structures</i>	134
4.2.7	<i>Spectra and structure functions near the wall</i>	139
4.3	Free shear flows	142
4.3.1	<i>Planar jets and wakes</i>	142
4.3.2	<i>The round jet</i>	148
4.4	Homogeneous shear flow	152
4.4.1	<i>The governing equations</i>	152
4.4.2	<i>The asymptotic state</i>	156
4.5	Heat transfer in wall-bounded shear flows—the log-law revisited	157
4.5.1	<i>Turbulent heat transfer near a surface and the log-law for temperature</i>	157
4.5.2	<i>The effect of stratification on the log-law—the atmospheric boundary layer</i>	163
4.6	More on one-point closure models	169
4.6.1	<i>A second look at the k-ϵ model</i>	169
4.6.2	<i>The Reynolds stress model</i>	176
4.6.3	<i>Large eddy simulation: a rival for one-point closures?</i>	180
5.	The phenomenology of Taylor, Richardson, and Kolmogorov	188
5.1	Richardson revisited	191
5.1.1	<i>Time and length scales in turbulence</i>	191
5.1.2	<i>The energy cascade pictured as the stretching of turbulent eddies</i>	195
5.1.3	<i>The dynamic properties of turbulent eddies: linear and angular impulse</i>	202
5.2	Kolmogorov revisited	211
5.2.1	<i>Dynamics of the small scales</i>	211
5.2.2	<i>Turbulence-induced fluctuations of a passive scalar</i>	221
5.3	The intensification of vorticity and the stretching of material lines	228
5.3.1	<i>Enstrophy production and the skewness factor</i>	228

5.3.2	<i>Sheets or tubes?</i>	231
5.3.3	<i>Examples of concentrated vortex sheets and tubes</i>	234
5.3.4	<i>Are there singularities in the vorticity field?</i>	236
5.3.5	<i>The stretching of material line elements</i>	240
5.3.6	<i>The interplay of the strain and vorticity fields</i>	243
5.4	Turbulent diffusion by continuous movements	252
5.4.1	<i>Taylor diffusion of a single particle</i>	254
5.4.2	<i>Richardson's law for the relative diffusion of two particles</i>	257
5.4.3	<i>The influence of mean shear on turbulent dispersion</i>	260
5.5	Why turbulence is never Gaussian	263
5.5.1	<i>The experimental evidence and its interpretation</i>	263
5.5.2	<i>A glimpse at closure schemes which assume near-Gaussian statistics</i>	267
5.6	Closure	268

Part 2 Freely decaying, homogeneous turbulence

6.	Isotropic turbulence (in real space)	275
6.1	Introduction: exploring isotropic turbulence in real space	275
6.1.1	<i>Deterministic cartoons versus statistical phenomenology</i>	276
6.1.2	<i>The strengths and weaknesses of Fourier space</i>	281
6.1.3	<i>An overview of this chapter</i>	283
6.2	The governing equations of isotropic turbulence	293
6.2.1	<i>Some kinematics: velocity correlation functions and structure functions</i>	293
6.2.2	<i>More kinematics: the simplifications of isotropy and the vorticity correlation function</i>	300
6.2.3	<i>A summary of the kinematic relationships</i>	304
6.2.4	<i>Dynamics at last: the Karman–Howarth equation</i>	308
6.2.5	<i>Kolmogorov's four-fifths law</i>	310
6.2.6	<i>The skewness factor and enstrophy production (reprise)</i>	312
6.2.7	<i>The dynamical equation for the third-order correlations and the problem of closure</i>	313
6.2.8	<i>Closure of the dynamical equations in the equilibrium range</i>	314
6.2.9	<i>Quasi-normal-type closure schemes (part 1)</i>	316
6.2.10	<i>Passive scalar mixing in isotropic turbulence and Yaglom's four-thirds law</i>	318
6.3	The dynamics of the large scales	320
6.3.1	<i>The classical view: Loitsyansky's integral and Kolmogorov's decay laws</i>	322
6.3.2	<i>Landau's angular momentum</i>	323
6.3.3	<i>Batchelor's pressure forces</i>	328
6.3.4	<i>Saffman's spectrum</i>	333
6.3.5	<i>A consistent theory of the large scales in Batchelor turbulence</i>	342
6.3.6	<i>A summary of the dynamics of the large scales</i>	346
6.4	The characteristic signature of eddies of different shape	348
6.4.1	<i>Townsend's model eddy and its relatives</i>	349
6.4.2	<i>Turbulence composed of Townsend's model eddies of different sizes</i>	353
6.4.3	<i>Other model eddies</i>	356

6.5	Intermittency in the inertial-range eddies	357
6.5.1	<i>A problem for Kolmogorov's theory?</i>	358
6.5.2	<i>The log-normal model of intermittency</i>	360
6.5.3	<i>The $\hat{\beta}$ model of intermittency</i>	363
6.6	Measuring the distribution of energy and enstrophy across the different eddy sizes	366
6.6.1	<i>A real-space function which represents, approximately, the variation of energy with scale</i>	366
6.6.2	<i>Relating energy distributions in real and Fourier space</i>	374
6.6.3	<i>Cascade dynamics in real space</i>	379
7.	The role of numerical simulations	394
7.1	What is DNS or LES?	394
7.1.1	<i>Direct numerical simulations (DNS)</i>	394
7.1.2	<i>Large eddy simulations (LES)</i>	398
7.2	On the dangers of periodicity	404
7.3	Structure in chaos	406
7.3.1	<i>Tubes, sheets, and cascades</i>	407
7.3.2	<i>On the taxonomy of worms and clusters of worms</i>	412
8.	Isotropic turbulence (in spectral space)	419
8.1	Kinematics in spectral space	420
8.1.1	<i>The Fourier transform and its properties</i>	421
8.1.2	<i>The Fourier transform as a filter</i>	425
8.1.3	<i>The autocorrelation function and power spectrum</i>	427
8.1.4	<i>The transform of the correlation tensor and the three-dimensional energy spectrum</i>	431
8.1.5	<i>One-dimensional energy spectra in three-dimensional turbulence</i>	434
8.1.6	<i>Relating the energy spectrum to the second-order structure function</i>	438
8.1.7	<i>A footnote: singularities in the spectrum due to anisotropy</i>	440
8.1.8	<i>Another footnote: the transform of the velocity field</i>	441
8.1.9	<i>Definitely the last footnote: what do $E(k)$ and $E_1(k)$ really represent?</i>	442
8.2	Dynamics in spectral space	445
8.2.1	<i>An evolution equation for $E(k)$</i>	445
8.2.2	<i>Closure in spectral space</i>	448
8.2.3	<i>Quasi-normal type closure schemes (part 2)</i>	454
 Part 3 Special topics		
9.	The influence of rotation, stratification, and magnetic fields on turbulence	467
9.1	The importance of body forces in geophysics and astrophysics	467
9.2	The influence of rapid rotation and stable stratification	470
9.2.1	<i>The Coriolis force</i>	470
9.2.2	<i>The Taylor-Proudman theorem</i>	473
9.2.3	<i>Properties of inertial waves</i>	474
9.2.4	<i>Turbulence in rapidly rotating systems</i>	479
9.2.5	<i>From rotation to stratification (or from cigars to pancakes)</i>	485

9.3	The influence of magnetic fields I: the MHD equations	488
9.3.1	<i>The interaction of moving conductors and magnetic fields: a qualitative overview</i>	489
9.3.2	<i>From Maxwell's equations to the governing equations of MHD</i>	494
9.3.3	<i>Simplifying features of low magnetic Reynolds number MHD</i>	498
9.3.4	<i>Simple properties of high magnetic Reynolds number MHD</i>	499
9.4	The influence of magnetic fields II: MHD turbulence	503
9.4.1	<i>The growth of anisotropy in MHD turbulence</i>	505
9.4.2	<i>The evolution of eddies at low magnetic Reynolds number</i>	507
9.4.3	<i>The Landau invariant for homogeneous MHD turbulence</i>	513
9.4.4	<i>Decay laws at low magnetic Reynolds number</i>	514
9.4.5	<i>Turbulence at high magnetic Reynolds number</i>	516
9.4.6	<i>The shaping of eddies by combined Coriolis and Lorentz forces</i>	520
9.5	Turbulence in the core of the earth	522
9.5.1	<i>An introduction to planetary dynamo theory</i>	523
9.5.2	<i>Numerical simulations of the geodynamo</i>	529
9.5.3	<i>Various cartoons of the geodynamo</i>	532
9.5.4	<i>An α^2 model of the geodynamo based on inertial wave packets</i>	538
9.6	Turbulence near the surface of the sun	553
10.	Two-dimensional turbulence	559
10.1	The classical picture of 2D turbulence: Batchelor's self-similar spectrum, the inverse energy cascade, and the $E(k) \sim k^{-3}$ enstrophy flux law	560
10.1.1	<i>What is two-dimensional turbulence?</i>	560
10.1.2	<i>What does the turbulence remember?</i>	565
10.1.3	<i>Batchelor's self-similar spectrum</i>	565
10.1.4	<i>The inverse energy cascade of Batchelor and Kraichnan</i>	567
10.1.5	<i>Different scales in 2D turbulence</i>	570
10.1.6	<i>The shape of the energy spectrum and the k^{-3} law</i>	571
10.1.7	<i>Problems with the k^{-3} law</i>	574
10.2	Coherent vortices: a problem for the classical theories	577
10.2.1	<i>The evidence</i>	577
10.2.2	<i>The significance</i>	579
10.3	The governing equations in statistical form	581
10.3.1	<i>Correlation functions, structure functions, and the energy spectrum</i>	582
10.3.2	<i>The two-dimensional Karman–Howarth equation</i>	586
10.3.3	<i>Four consequences of the Karman–Howarth equation</i>	586
10.3.4	<i>The two-dimensional Karman–Howarth equation in spectral space</i>	588
10.4	Variational principles for predicting the final state in confined domains	591
10.4.1	<i>Minimum enstrophy</i>	592
10.4.2	<i>Maximum entropy</i>	594
10.5	Quasi-two-dimensional turbulence: bridging the gap with reality	594
10.5.1	<i>The governing equations for shallow-water, rapidly rotating flow</i>	595
10.5.2	<i>The Karman–Howarth equation for shallow-water, rapidly rotating turbulence</i>	597

Epilogue	601
Appendix 1: Vector identities and an introduction to tensor notation	603
A1.1 Vector identities and theorems	603
A1.2 An introduction to tensor notation	606
Appendix 2: The properties of isolated vortices: invariants, far-field properties, and long-range interactions	611
A2.1 The far-field velocity induced by an isolated eddy	611
A2.2 The pressure distribution in the far field	613
A2.3 Integral invariants of an isolated eddy: linear and angular impulse	614
A2.4 Long-range interactions between eddies	617
Appendix 3: Hankel transforms and hypergeometric functions	620
A3.1 Hankel transforms	620
A3.2 Hypergeometric functions	621
Appendix 4: The kinematics of homogeneous, axisymmetric turbulence	623
<i>Subject index</i>	625