

Contents

<i>List of Figures</i>	<i>page</i> xiii
<i>Foreword by Robert J. Aumann</i>	xv
<i>Preface</i>	xxi
<i>Acknowledgments</i>	xxiii
<i>Presentation of the Content</i>	xxv

Part A: Background Material

I	Basic Results on Normal Form Games	3
I.1	The Minmax Theorem	4
I.1.a	Definitions and Notations	4
I.1.b	A Basic Theorem	5
I.1.c	Convexity	6
I.1.d	Mixed Strategies	6
I.1.e	Note on the Separation Theorem	9
Exercises		10
I.2	Complements to the Minmax Theorem	15
I.2.a	The Topology on S	16
I.2.b	Lack of Continuity: Regularization	16
I.2.c	Lack of Compactness: Approximation	19
I.2.d	Measurability: Symmetric Case	20
I.2.e	Pure Optimal Strategies	23
Exercises		24
I.3	The Minmax Theorem for Ordered Fields	31
Exercises		32
I.4	Equilibrium Points	45
Exercises		45

II	Basic Results on Extensive Form Games	58
II.1	The Extensive Form	58
II.1.a	Definitions	58
II.1.b	The Finite Case	60
II.1.c	A Measurable Setup	62
	Exercises	64
II.2	Infinite Games	90
II.2.a	Infinite Games with Perfect Information	90
II.2.b	Remarks: Infinite Games without Perfect Information	95
	Exercises	96
II.3	Correlated Equilibria and Extensions	101
II.3.a	Correlated Equilibria	101
II.3.b	Multistage Games, Extensive Form Correlated Equilibria	103
II.3.c	Communication Equilibria	105
II.3.d	Finite Games	108
	Exercises	110
II.4	Vector Payoffs	118
	Exercises	121
III	The Belief Space	123
III.1	The Universal Belief Space	123
III.1.a	States of the World and Types	124
III.1.b	Belief Subspaces	135
III.2	Consistency and Common Knowledge	138
III.3	An Approximation Theorem	151
III.4	Games with Incomplete Information	153
III.4.a	The Model	153
III.4.b	Two-Person Zero-Sum Case	154
III.4.c	“Approachability” in One-Shot Games	158
III.4.d	Concavification and Convexification	161
	Exercises	166
IV	General Model of Repeated Games	171
IV.1	The Model	171
IV.1.a	States, Signals, and Transitions	172
IV.1.b	Strategies and Payoffs	173
IV.1.c	Zero-Sum Case	174
IV.1.d	Non-Zero-Sum Case	176
IV.1.e	Stochastic Games and Games with Incomplete Information	177
	Exercises	178

IV.2	Equivalent Representations	178
IV.2.a	Simple Transformations	178
IV.2.b	A Deterministic Framework	180
IV.2.c	A Combinatorial Form	182
IV.3	Recursive Structure	183
IV.3.a	A Canonical Representation	183
IV.3.b	The Recursive Formula	184
	Exercises	187
IV.4	Supergames	190
IV.4.a	Standard Signaling	190
IV.4.b	Partial Monitoring	193
	Exercises	201
IV.5	Recursive Games	205
	Exercises	208
 Part B: The Central Results		
V	Full Information on One Side	215
V.1	General Properties	215
V.2	Elementary Tools and the Full Monitoring Case	217
V.2.a	Posterior Probabilities and Non-Revealing Strategies	218
V.2.b	$\lim v_n(p)$ and $v_\infty(p)$	222
V.2.c	Approachability Strategy	224
V.3	The General Case	225
V.3.a	$\lim v_n(p)$ and $v_\infty(p)$	226
V.3.b	The Non-Revealing Game	227
V.3.c	Study of $v_\infty(p)$	230
V.3.d	Optimal Strategy for the Uninformed Player	233
V.3.e	Approachability	239
V.3.f	The Errors E_n^+ in the Approachability Theorem	248
V.3.g	Implications of the Approachability Theorem	257
V.3.h	A Continuum of Types	259
V.3.i	Implications of the Approachability Theorem (continued)	270
V.4	The Role of the Normal Distribution	279
V.4.a	The Heuristics of the Result	282
V.4.b	Proof of Theorem V.4.1	284
V.4.c	More General Results	291
V.5	The Speed of Convergence of v_n	294
V.5.a	State-Independent Signaling	294
V.5.b	State-Dependent Signaling	296
V.5.c	Games with Error Term $\approx (\ln n)/n$	299

Exercises	302
V.6 Appendix	316
VI Incomplete Information on Two Sides	326
VI.1 Introduction	326
VI.2 General Preparations	326
VI.2.a Definitions and Notations	326
VI.2.b Preliminary Results	328
VI.2.c An Auxiliary Game	330
VI.2.d The Probabilistic Structure	331
VI.3 The Infinite Game	339
VI.3.a Minmax and Maxmin	339
VI.3.b Approachability	345
VI.4 The Limit of $v_n(p)$	357
VI.5 The Functional Equations: Existence and Uniqueness	362
VI.6 On the Speed of Convergence of v_n	365
VI.7 Examples	366
Exercises	379
VII Stochastic Games	392
VII.1 Discounted Case	392
VII.1.a Zero-Sum Case	393
VII.1.b Non-Zero-Sum Case (Finite)	395
VII.1.c Non-Zero-Sum Case (General)	396
VII.2 Asymptotic Analysis, Finite Case: The Algebraic Aspect	399
VII.3 ε -Optimal Strategies in the Undiscounted Game	401
VII.3.a The Theorem	401
VII.3.b Proof of the Theorem under $H(L, \lambda, A, \delta)$	402
VII.3.c End of the Proof	404
VII.3.d Particular Cases (Finite Games, Two-Person Zero-Sum)	407
VII.4 The Two-Person Non-Zero-Sum Undiscounted Case	410
VII.4.a An Example	410
VII.4.b Games with Absorbing States	413
Exercises	416
VII.5 Reminder about Dynamic Programming	424
Exercises	425
Part C: Further Developments	
VIII Extensions and Further Results	431
VIII.1 Incomplete Information: The Symmetric Case	431
VIII.2 Games with No Signals	433
VIII.2.a Presentation	433

	VIII.2.b	An Auxiliary Game	434
	VIII.2.c	Minmax and Maxmin	436
	VIII.2.d	$\lim v_n$ and $\lim v_\lambda$	443
VIII.3		A Game with State-Dependent Signaling Matrices	446
	VIII.3.a	Introduction and Notation	446
	VIII.3.b	Minmax	447
	VIII.3.c	Maxmin	452
VIII.4		Stochastic Games with Incomplete Information	457
	VIII.4.a	A First Class	458
	VIII.4.b	A Second Class	464
	VIII.4.c	Minmax: Two More Examples	469
		Exercises	474
IX		Non-Zero-Sum Games with Incomplete Information	481
	IX.1	Equilibria in Γ_∞	481
		IX.1.a Existence	481
		IX.1.b Characterization (Hart, 1985)	484
	IX.2	Bi-Convexity and Bi-Martingales	492
	IX.3	Correlated Equilibrium and Communication Equilibrium	495
		IX.3.a Communication Equilibrium	496
		IX.3.b "Noisy Channels"; Characterization of D_r , ($0 < r < \infty$)	504
		Exercises	507
		Appendix A: Reminder about Analytic Sets	511
	A.1	Notation	511
	A.2	Souslin Schemes	511
	A.3	K -Analytic and K -Lusin Spaces	512
	A.4	Capacities	513
	A.5	Polish, Analytic, and Lusin Spaces	515
	A.6	Blackwell Spaces and Standard Borel Spaces	517
	A.7	Spaces of Subsets	518
	A.8	Some Harder Results	519
	A.9	Complements to Measure Theory	520
	A.10	*-Radon Spaces	522
		Appendix B: Historical Notes	526
		Chapter I	526
		Chapter II	527
		Chapter III	528
		Chapter IV	529
		Chapter V	529
		Chapter VI	530

Chapter VII	530
Chapter VIII	531
Chapter IX	531
Appendix C: Bibliography	533
Appendix D: Updates	548
D.1 Complements and Advances	548
D.2 Complementary Bibliography	552
<i>Author Index</i>	561
<i>Subject Index</i>	564