

# Contents

<b>1</b>	<b>Introductory Ideas</b> .....	1
1.1	The Bergman Kernel .....	1
1.1.1	Calculating the Bergman Kernel .....	14
1.1.2	The Poincaré-Bergman Distance on the Disc.....	19
1.1.3	Construction of the Bergman Kernel by Way of Differential Equations .....	20
1.1.4	Construction of the Bergman Kernel by Way of Conformal Invariance .....	23
1.2	The Szegő and Poisson–Szegő Kernels .....	25
1.3	Formal Ideas of Aronszajn .....	35
1.4	A New Bergman Basis .....	36
1.5	Further Examples .....	39
1.6	A Real Bergman Space .....	40
1.7	The Behavior of the Singularity in a General Setting .....	41
1.8	The Annulus .....	43
1.9	A Direct Connection Between the Bergman and Szegő Kernels ....	45
1.9.1	Introduction .....	45
1.9.2	The Case of the Disc .....	45
1.9.3	The Unit Ball in $\mathbb{C}^n$ .....	49
1.9.4	Strongly Pseudoconvex Domains .....	52
1.9.5	Concluding Remarks.....	53
1.10	Multiply Connected Domains .....	53
1.11	The Bergman Kernel for a Sobolev Space.....	54
1.12	Ramadanov’s Theorem .....	56
1.13	Coda on the Szegő Kernel .....	58
1.14	Boundary Localization .....	59
1.14.1	Definitions and Notation.....	60
1.14.2	A Representative Result .....	60

1.14.3	The More General Result in the Plane .....	62
1.14.4	Domains in Higher-Dimensional Complex Space .....	62
	Exercises .....	65
<b>2</b>	<b>The Bergman Metric</b> .....	<b>71</b>
2.1	Smoothness to the Boundary of Biholomorphic Mappings .....	71
2.2	Boundary Behavior of the Bergman Metric .....	81
2.3	The Biholomorphic Inequivalence of the Ball and the Polydisc ....	83
	Exercises .....	84
<b>3</b>	<b>Further Geometric and Analytic Theory</b> .....	<b>87</b>
3.1	Bergman Representative Coordinates .....	87
3.2	The Berezin Transform .....	90
3.2.1	Preliminary Remarks .....	90
3.2.2	Introduction to the Poisson–Bergman Kernel .....	91
3.2.3	Boundary Behavior .....	94
3.3	Ideas of Fefferman .....	98
3.4	Results on the Invariant Laplacian .....	100
3.5	The Dirichlet Problem for the Invariant Laplacian on the Ball .....	109
3.6	Concluding Remarks .....	115
	Exercises .....	115
<b>4</b>	<b>Partial Differential Equations</b> .....	<b>117</b>
4.1	The Idea of Spherical Harmonics .....	117
4.2	Advanced Topics in the Theory of Spherical Harmonics: The Zonal Harmonics .....	117
4.3	Spherical Harmonics in the Complex Domain and Applications....	130
4.4	An Application to the Bergman Projection .....	141
	Exercises .....	145
<b>5</b>	<b>Further Geometric Explorations</b> .....	<b>147</b>
5.1	Introductory Remarks .....	147
5.2	Semicontinuity of Automorphism Groups .....	151
5.3	Convergence of Holomorphic Mappings .....	156
5.3.1	Finite Type in Dimension Two .....	156
5.4	The Semicontinuity Theorem .....	166
5.5	Some Examples .....	168
5.6	Further Remarks .....	168
5.7	The Lu Qi-Keng Conjecture .....	169
5.8	The Lu Qi-Keng Theorem .....	171
5.9	The Dimension of the Bergman Space .....	174
5.10	The Bergman Theory on a Manifold .....	178
5.10.1	Kernel Forms .....	178
5.10.2	The Invariant Metric .....	182
5.11	Boundary Behavior of the Bergman Metric .....	184
	Exercises .....	185

<b>6</b>	<b>Additional Analytic Topics</b> .....	187
6.1	The Diederich–Fornæss Worm Domain .....	187
6.2	More on the Worm .....	192
6.3	Non-Smooth Versions of the Worm Domain .....	199
6.4	Irregularity of the Bergman Projection .....	200
6.5	Irregularity Properties of the Bergman Kernel .....	205
6.6	The Kohn Projection Formula .....	207
6.7	Boundary Behavior of the Bergman Kernel .....	208
	6.7.1 Hörmander’s Result on Boundary Behavior .....	209
	6.7.2 The Fefferman’s Asymptotic Expansion .....	215
6.8	The Bergman Kernel for a Sobolev Space .....	221
6.9	Regularity of the Dirichlet Problem on a Smoothly Bounded Domain and Conformal Mapping .....	224
6.10	Existence of Certain Smooth Plurisubharmonic Defining Functions for Strictly Pseudoconvex Domains and Applications ...	228
	6.10.1 Introduction .....	228
6.11	Proof of Theorem 6.10.1 .....	229
6.12	Application of the Complex Monge–Ampère Equation .....	233
6.13	An Example of David Barrett .....	235
6.14	The Bergman Kernel as a Hilbert Integral .....	245
	Exercises .....	249
<b>7</b>	<b>Curvature of the Bergman Metric</b> .....	251
7.1	What is the Scaling Method? .....	251
7.2	Higher Dimensional Scaling .....	252
	7.2.1 Nonisotropic Scaling .....	252
	7.2.2 Normal Convergence of Sets .....	254
	7.2.3 Localization .....	255
7.3	Klembeck’s Theorem with $C^2$ -Stability .....	261
	7.3.1 The Main Goal .....	261
	7.3.2 The Bergman Metric near Strictly Pseudoconvex Boundary Points .....	262
	Exercises .....	263
<b>8</b>	<b>Concluding Remarks</b> .....	273
	<b>Table of Notation</b> .....	275
	<b>Bibliography</b> .....	277
	<b>Index</b> .....	287