

Contents

	<i>page</i>
<i>Preface</i>	xi
1 Introduction	1
1.1 Trigonometric polynomials and series	1
1.2 The dawn of the theory	4
1.2.1 The vibrating string controversy	5
1.2.2 Fourier's view on heat flow	11
1.3 Application: irrationality of π	12
1.4 Exercises	13
1.4.1 Statements	13
1.4.2 Hints	14
1.4.3 Solutions	14
2 The Lebesgue measure and integral	15
2.1 Historical considerations	15
2.1.1 Ancient measure theory	15
2.1.2 The Riemann integral	18
2.1.3 Outer measure and measurability	22
2.2 A brief outline of the Lebesgue integral	28
2.2.1 Definition and basic properties	28
2.2.2 Multiple integrals	35
2.2.3 The anti-derivative problem	38

2.2.4	Length of curves	43
2.3	Abstract measure theory	47
2.4	Exercises	51
2.4.1	Statements	51
2.4.2	Hints	57
2.4.3	Solutions	59
2.5	Notes to Chapter 2	68
3	Elements of functional analysis	71
3.1	An overall perspective	71
3.2	Hilbert spaces	73
3.3	Banach spaces	83
3.4	Functionals and operators	93
3.4.1	The family of bounded linear operators	94
3.4.2	The Hahn–Banach theorems	99
3.4.3	Baire category and consequences	108
3.4.4	The spectral theorem	112
3.5	Fréchet spaces	124
3.6	Exercises	127
3.6.1	Statements	128
3.6.2	Hints	137
3.6.3	Solutions	140
3.7	Notes to Chapter 3	158
4	Convergence results for Fourier series	159
4.1	Basic properties of Fourier coefficients	163
4.2	Pointwise convergence	165
4.3	Mean-square convergence	176
4.4	Convergence at a jump discontinuity	177
4.5	Exercises	182
4.5.1	Statements	182

4.5.2	Hints	186
4.5.3	Solutions	187
4.6	Notes to Chapter 4	197
5	Fourier transforms	199
5.1	Rapidly decreasing smooth functions	201
5.2	Fourier transform for square integrable functions	205
5.3	Fourier transform for integrable functions	206
5.4	Exercises	208
5.4.1	Statements	208
5.4.2	Hints	210
5.4.3	Solutions	211
5.5	Note to Chapter 5	216
6	Multi-dimensional Fourier analysis	217
6.1	Fourier transform	217
6.1.1	The class of tempered distributions	220
6.1.2	Fourier transform of a tempered distribution	227
6.2	Fourier series	231
6.2.1	L^2 -convergence	232
6.2.2	Pointwise convergence	232
6.2.3	The tempered distributions approach	235
6.3	Fourier transform of a measure	241
6.4	The Fourier transform on $L^p(\mathbb{R})$ -spaces	246
6.5	Sobolev spaces	247
6.6	Periodic Sobolev spaces	254
6.7	Exercises	254
6.7.1	Statements	254
6.7.2	Hints	258
6.7.3	Solutions	260
6.8	Notes to Chapter 6	271

7	A glance at some advanced topics	273
7.1	Complex analysis techniques	273
7.1.1	Basic facts about analytic functions	273
7.1.2	Fourier series convergence by change of variables	311
7.1.3	Paley–Wiener theorems	312
7.1.4	Hardy spaces	317
7.2	Pseudodifferential operators	329
<i>Afterword</i>		335
Appendix	Historical notes	337
<i>References</i>		343
<i>Index</i>		349