Contents

1 Introduction **1**

1.1 What is a boundary problem? 1
1.2 Classification of partial differential equations 12
1.3 Types of boundary conditions and the collocation method . . . 18
1.4 Differential equations as models for nature 22

2 Boundary problems of ordinary differential equations **27**

2.1 Linear differential equations 27
2.2 Solving linear differential equations 31
2.3 Differential equations of physics and engineering 44
2.4 Boundary value problems and eigenvalues 51
2.5 Boundary value problems as initial value problems 59
2.6 Nonlinear ordinary differential equations 68
2.7 Solutions of nonlinear differential equations 80

3 Partial differential equations **91**

3.1 Coordinate systems and separability 91
3.2 Methods to reduce partial to ordinary differential equations . . 104
3.3 The method of characteristics 116
3.4 Nonlinear partial differential equations 126

4 Boundary problems with one closed boundary **137**

4.1 LAPLACE and POISSON equations 137
4.2 Conformal mapping in two and three dimensions 156
4.3 D'ALEMBERT wave equation and string vibrations 168
4.4 HELMHOLTZ equation and membrane vibrations 174
4.5 Rods and the plate equation 188
4.6 Approximation methods 202
4.7 Variational calculus 214
4.8 Collocation methods 223

5 Boundary problems with two closed boundaries **241**

5.1 Inseparable problems 241
5.2 Holes in the domain. Two boundaries belonging to different coordinate systems 263
5.3 Corners in the boundary 279

6 Nonlinear boundary problems **285**
6.1 Some definitions and examples 285
6.2 Moving and free boundaries 288
6.3 Waves of large amplitudes. Solitons 295
6.4 The rupture of an embankment-type water dam 306
6.5 Gas flow with combustion 310

References **313**

Appendix **325**

Index **334**