

Contents

1	Introduction	1
2	Preliminaries	5
2.1	Notation	5
2.2	Bayesian Nash Equilibrium	6
2.3	Auctions as Games	9
2.3.1	What is an Auction?	9
2.3.2	Auction Types	10
2.3.3	Auction as Bayesian Games	11
3	Private Values	13
3.1	The Independent Private Values Model	13
3.1.1	First-price Auctions	14
3.1.2	Second-price Auctions	18
3.1.3	Revenue Equivalence	20
3.1.4	Reserve Prices and Entry Fees	22
3.2	The Correlated Private Values Model	25
3.2.1	Second-price Auction	26
3.2.2	First-price Auction	27
3.2.3	Comparison of Expected Payment	29
3.3	The Effect of Risk Aversion	32
3.3.1	Revenue Comparison	33
3.4	The Discrete Valuation Case	34
3.5	Exercises	35
4	Common Value	39
4.1	An Example with Independent Signals	40
4.1.1	First-price Auction	40
4.1.2	Second-price Auction Example	42
4.2	An Example with Correlated Types	42
4.2.1	First-price Auction	43
4.2.2	Second-price Auction	45
4.3	The Symmetric Model with Two Bidders	46

4.3.1	Second-price Auctions	50
4.3.2	First-price Auctions	51
4.3.3	Revenue Comparison	54
4.4	Exercises	55
5	Affiliated Values	57
5.1	The General Model	58
5.2	Second-price Auctions	63
5.3	First-price Auctions	65
5.4	English Auctions	67
5.5	Expected Revenue Ranking	68
5.6	The Effects of Risk Aversion	70
5.7	Exercises	73
6	Mechanism Design	75
6.1	The Revelation Principle	75
6.2	Direct Mechanisms	79
6.3	Revenue Equivalence and the Optimal Auction	83
6.4	Some Extensions	89
6.4.1	Non-monotonic Marginal Valuation	89
6.4.2	Correlated Values	101
6.4.3	Several Objects	115
6.4.4	Common Values Auction	117
6.5	Exercises	119
7	Multiple Objects	121
7.1	Sequential Auctions	122
7.2	Simultaneous Auctions	124
7.2.1	Discriminatory Auctions	125
7.2.2	Uniform price Auctions	130
7.3	Optimal Auction	137
7.4	Exercises	143
A	Probability	145
A.1	Probability Spaces	145
A.2	Uncountable Sample Space Case	147
A.3	Random Variables	148
A.4	Random Vectors and their Distribution	150
A.5	Independence of Random Variables	152
A.6	The Distribution of the Maximum of Independent Random Variables	152
A.7	The Distribution of the Second Highest Value	153
A.8	Mean Value of Random Variables	154
A.9	Conditional Probability	156

B Differential Equations	159
B.1 The Simplest Differential Equation	159
B.2 Integrating Factor	159
C Affiliation	161
D Convexity	165
References	169
Index of Notations	173
Index of Proper Names	175
Index	177