Contents

Cont Prefa	ribu Ice	itors	xv xix		
1.	Diffusion operators for multimodal data analysis Tal Shnitzer, Roy R. Lederman, Gi-Ren Liu,				
	Ro	onen Talmon, and Hau-Tieng Wu			
	1	Introduction	2		
	2	Preliminaries: Diffusion Maps	3		
	3	Alternating Diffusion	5		
		3.1 Problem formulation: Metric spaces and			
		probabilistic setting	5		
		3.2 Illustrative example	5		
		3.3 Algorithm	10		
		3.4 Common manifold interpretation	12		
	4	Self-adjoint operators for recovering hidden components	17		
		4.1 Problem formulation	18		
		4.2 Operator definition and analysis	19		
		4.3 Discrete setting	22		
	5	Applications	23		
		5.1 Shape analysis	23		
		5.2 Foetal heart rate recovery	25		
		5.3 Sleep dynamics assessment	27		
	Re	ferences	38		
2.	In	trinsic and extrinsic operators for shape analysis	41		
	Yι	i Wang and Justin Solomon			
	1	Introduction	42		
	2	Preliminaries	44		
		2.1 Extrinsic and intrinsic geometry	44		
		2.2 Operators and spectra	45		
	3	Theoretical aspects and numerical analysis	45		
		3.1 Basics of linear operators	45		
		3.2 PDEs and Green's functions	49		
		3.3 Operator derivation and discretization	52		
		3.4 Operators and geometry	55		
		3.5 Inverse problems	56		

	4	Spectral shape analysis and applications			
		4.1	Spectral analysis: From Euclidean space to manifold	58	
		4.2	Spectral data analysis	59	
		4.3	Spectral analysis: Point embedding, signature,		
			and geometric descriptors	62	
		4.4	Shape analysis and geometry processing	66	
		4.5	Other aspects of spectral shape analysis	72	
		4.6	Numerical aspects	73	
	5	Rele	vant geometric operators	76	
		5.1	Identity operator, area form, and mass matrix	76	
		5.2	Laplace–Beltrami (intrinsic Laplacian)	78	
		5.3	Combinatorial and graph Laplacians	80	
		5.4	Restricted Laplacian	80	
		5.5	Scale invariant Laplacian	81	
		5.6	Affine and equi-affine invariant Laplacian	81	
		5.7	Anisotropic Laplacian	82	
		5.8	Hessian and normal-restricted Hessian: A family		
			of linearized energies	83	
		5.9	Modified Dirichlet energy	83	
		5.10	Hamiltonian operator and Schrödinger operator	84	
		5.11	Curvature Laplacian	85	
		5.12	Concavity-aware Laplacian	85	
		5.13	Extrinsic and relative Dirac operators	86	
		5.14	Intrinsic Dirac operator D_1	88	
		5.15	Volumetric (extrinsic) Laplacian	88	
		5.16	Hessian energy	89	
		5.17	Single layer potential operator and kernel method	90	
		5.18	Dirichlet-to-Neumann operator (Poincaré–Steklov		
			operator)	91	
		5.19	Other extrinsic methods	92	
	6	Sum	mary and experiments	92	
		6.1	Experiments	93	
		6.2	Eigenfunctions	93	
		6.3	Heat kernel signatures	96	
		6.4	Segmentation	97	
	_	6.5	Distance or dissimilarity	100	
	7	Cone	clusion and future work	103	
		7.1	Summary	103	
		7.2	Future work	104	
	AC	Acknowledgements			
	Ke	teren	ces	105	
3.	0	pera	tor-based representations of discrete	117	
	ta	ngen	it vector neids	117	
	M	irela B	en-Chen and Omri Azencot		
	1	Intro	duction	119	
		1.1	Organization	120	

	2	Smooth functional vector fields					
		2.1 Notation	120				
		2.2 Directional derivative of functions	121				
		2.3 Functional vector fields	121				
		2.4 Flow maps	122				
		2.5 Functional flow maps	123				
		2.6 Lie bracket	126				
	3	Discrete functional vector fields	128				
		3.1 Notation	128				
		3.2 Directional derivative of functions	131				
		3.3 Functional vector fields	131				
		3.4 Flow maps	133				
		3.5 Functional flow maps	133				
		3.6 Lie bracket	135				
	4	Divergence-based functional vector fields	136				
		4.1 Smooth DFVF	136				
		4.2 Discrete DFVF	136				
		4.3 Mixed Lie bracket operator	137				
		4.4 The Lie bracket as a linear transformation					
		on vector fields	142				
		4.5 Integrating the Lie bracket operator	144				
		4.6 Outlook	145				
	5	Conclusion and future work	146				
	Re	eferences	146				
4.	A	ctive contour methods on arbitrary graphs					
	ba	ased on partial differential equations	149				
	Cl	hristos Sakaridis, Nikos Kolotouros,					
	Ki	mon Drakopoulos, and Petros Maragos					
	1	Introduction	150				
	2	Rackground and related work	150				
	2	Active contours on graphs via geometric approximations	131				
	5	of aradient and curvature	155				
		2.1 Coometric gradient approximation on graphs	155				
		2.2 Coometric graviture approximation on graphs	155				
		2.2 Geometric curvature approximation on graphs	165				
	٨	Active contours on graphs using a finite element	100				
	4	fremowerk	160				
		I a Broklow formulation and numerical	109				
		4.1 FTODIETH TOTTINIATION and numerical	140				
		approximation	109				
	-	4.2 Locally constrained contour evolution	1/4				
	5 £	Conclusion	1/0				
	0 D~						
	ĸe	alerences	10/				

Fa in	Fast operator-splitting algorithms for variationalimaging models: Some recent developmentsRoland Glowinski, Shousheng Luo, and Xue-Cheng Tai				
Re					
1	Intr	oduction	193		
2	Reg	ularizers and associated variational models for image			
	restoration				
	2.1	Generalities	19		
	2.2	Total variation regularization	194		
	2.3	The Euler elastica regularization	19		
	2.4	L^1 -Mean curvature and L^1 -Gaussian curvature energies	19		
	2.5	Willmore bending energy	19		
	2.6	Summary	198		
3	Basi	c results, notations and an introduction			
	to o	operator-splitting methods	198		
	3.1	Basic results and notations	198		
	3.2	The Lie and Marchuk–Yanenko operator-splitting			
		schemes for the time discretization of initial value			
		problems	200		
	3.3	Time discretization of the initial value problem (17)			
		by the Lie scheme	20		
	3.4	Asymptotic properties of the Lie and Marchuk–Yanenko			
	-	schemes	202		
4	Operator splitting method for Euler elastica energy				
	fund	tional	203		
	4.1	Formulation of the problem and operator-splitting			
		solution methods	203		
	4.2	On the solution of problem (44)	207		
	4.3	On the solution of problem (45)	208		
	4.4	On the solution of problem (46)	210		
5	An o	operator-splitting method for the Willmore			
	ene	rgy-based variational model	210		
	5.1	Formulation of the problem and operator-splitting			
		solution methods	210		
	5.2	On the solution of problem (77)	213		
	5.3	On the solution of problem (78)	215		
	5.4	Estimating y	216		
6	And	operator-splitting method for the L^1 -mean curvature			
-	variational model				
7	Ope	rator-splitting methods for the ROF model	221		
-	7.1	Generalities: synopsis	221		
	7.2	A first operator-splitting method	222		
	7.3	A second operator-splitting method	274		
8	Con	clusion	227		
Ăr	know	ledgements	227		
Re	References				

6.	From active contours to minimal geodesic paths: New solutions to active contours problems by Fikonal equations						
	Da Chen and Laurent D. Cohen						
	1	Introduction					
		1.1 Outline	235				
	2	Active contour models	236				
		2.1 Edge-based active contour model	236				
		2.2 The piecewise smooth Mumford–Shah model and					
		the piecewise constant reduction model	240				
	3	Minimal paths for edge-based active contours problems	243				
		3.1 Cohen-Kimmel minimal path model	243				
		3.2 Finsler and Randers minimal paths	245				
	4	Minimal paths for alignment active contours	247				
		4.1 Randers alignment minimal paths	247				
		4.2 Riemannian alignment minimal paths	251				
	5	Orientation-lifted Randers minimal paths for Euler-Mumford					
		elastica problem	253				
		5.1 Euler–Mumford elastica problem and its Finsler metric					
		interpretation	254				
		5.2 Finsler elastica geodesic path for approximating					
		the elastica curve	255				
		5.3 Data-driven Finsler elastica metric	257				
	6	Randers minimal paths for region-based active contours	261				
		6.1 Hybrid active contour model	261				
		6.2 A Randers metric interpretation to the hybrid energy	263				
		6.3 Practical implementations	264				
	_	6.4 Application to image segmentation	265				
	7	/ Conclusion					
	Acknowledgements						
	Re	ferences	269				
7.	С	omputable invariants for curves and surfaces	273				
	0	hri Halimi. Dan Raviv Yonathan Aflalo					
	an	d Ron Kimmel					
	1	Introduction	274				
	2	Scale invariant metric	277				
		2.1 Scale invariant arc-length for planar curves	277				
		2.2 Scale invariant metric for implicitly defined planar curves	278				
		2.3 Scale invariant metric for surfaces	279				
		2.4 Approximating the scale invariant Laplace–Beltrami					
		operator for surfaces	280				
	3	Equi-affine invariant metric	285				
		3.1 Equi-affine invariant arc-length	285				
		3.2 Equi-affine metric for surfaces	286				

	4 Affine metric	288
	4.1 Affine invariant arc-length	288
	4.2 Affine metric for surfaces	289
	4.3 Approximating the Gaussian curvature	292
	5 Applications	294
	5.1 Self functional maps: A song of shapes and operators	294
	5.2 Object recognition	305
	6 Summary	310
	Acknowledgements	310
	References	311
8.	Solving PDEs on manifolds represented as point clouds and applications	315
	Rongjie Lai and Hongkai Zhao	
	1 Introduction	316
	2 Solving PDEs on manifolds represented as point clouds	318
	2.1 Moving least square methods	318
	2.2 Local mesh method	320
	3 Solving Fokker–Planck equation for dynamic system	324
	3.1 Double well potential	327
	3.2 Rugged Mueller potential	329
	4 Solving PDEs on incomplete distance data	331
	5 Geometric understanding of point clouds data	337
	5.1 Construction of skeletons from point clouds	338
	5.2 Construction of conformal mappings from point clouds	339
	5.3 Nonrigid manifolds registration using LB eigenmap	340
	6 Conclusion	343
	Acknowledgements	345
	References	345
9.	Tighter continuous relaxations for MAP inference	
	in discrete MRFs: A survey	351
	Hariprasad Kannan, Nikos Komodakis, and Nikos Paragios	
	1 LP relaxation	354
	1.1 Tightening the polytope	357
	2 Cluster pursuit algorithms	362
	3 Cycles in the graph	369
	3.1 Searching for frustrated cycles	372
	3.2 Efficient MAP inference in a cycle	374
	3.3 Planar subproblems	375
	4 Tighter subgraph decompositions	377
	4.1 Global higher-order cliques	378
	5 Semidefinite programming-based relaxation	380
	5.1 Rounding schemes for SDP relaxation	392
	5.2 Problems where SDP relaxation has helped	393
	· · · · · ·	

	6 Characterizing tight relaxations					
	7 Conclusion					
	Re	References				
10.	La	agrangian methods for composite optimization	401			
	Shoham Sabach and Marc Teboulle					
	1	Introduction	402			
	2	The Lagrangian framework	404			
		2.1 Lagrangian-based methods: Basic elements				
		and mechanism	404			
		2.2 Proximal mappings and minimization	407			
		2.3 Application examples	411			
	3	The convex setting	414			
		3.1 Preliminaries on the convex model (CM)	414			
		3.2 Proximal method of multipliers and fundamental				
		Lagrangian-based schemes	416			
		3.3 One scheme for all: A perturbed PMM and its global	410			
		rate analysis	418			
		3.4 Special cases of the perturbed PMIM: Fundamental	433			
	Λ	The non-convex setting	422			
	4	4.1 The nonconvex poplinear composite optimization	424			
		Preliminaries	425			
		4.2 ALBUM—Adaptive Lagrangian-based multiplier method	427			
		4.3 A methodology for global analysis of Lagrangian-based	427			
		methods	429			
		4.4 ALBUM in action: Global convergence of	,			
		Lagrangian-based schemes	431			
	Ac	knowledgements	433			
	Re	eferences	433			
11.	G	enerating structured nonsmooth priors and				
	as	sociated primal-dual methods	437			
	М	ichael Hintermüller and Kostas Papafitsoros				
	1	Introduction	438			
		1.1 Context	438			
		1.2 Main contributions and organization of this chapter	449			
	2	Nonsmooth priors	449			
		2.1 Total Variation	449			
		2.2 Total generalized variation	453			
		2.3 Dualization	455			
		2.4 Dualization of the variational regularization problems	461			
	3	Numerical algorithms	462			
	4	Bilevel optimization	469			
		4.1 Background	469			
		4.2 Bilevel optimization—A monolithic approach	473			

	5 Numerical examples	480
	5.1 Discrete operators for (\mathbb{P}_{TV})	481
	5.2 Bilevel TV numerical experiments	485
	5.3 Discrete operators for (\mathbb{P}_{TGV})	488
	5.4 Bilevel TGV numerical experiments	491
	References	494
12.	Graph-based optimization approaches for machine learning, uncertainty quantification and networks	503
	Andrea L. Bertozzi and Ekaterina Merkurjev	
	1 Introduction	504
	2 Graph theory	505
	3 Recent methods for semisupervised and unsupervised	
	data classification	507
	3.1 Semisupervised learning and the Ginzburg–Landau	
	graph model	508
	3.2 The graph MBO scheme for data classification and	F11
	image processing	511
	3.4 Unsupervised learning and the Mumford Shah model	513
	3.5 Imposing volume constraints	515
	4 Total variation methods for semisupervised and unsupervised	515
	data classification	518
	5 Uncertainty quantification within the graphical framework	522
	6 Networks	523
	7 Conclusion	526
	Acknowledgements	527
	References	527
13.	Survey of fast algorithms for Euler's elastica-based	522
	image segmentation	533
	Sung Ha Kang, Xue-Cheng Tai, and Wei Zhu	•
	1 Introduction	534
	2 Piecewise constant representation and interface problems	
	to illusory contour with curvature term	535
	3 Euler's elastica-based segmentation models and fast algorithms	540
	4 Discussion	547
	References	548
14.	Recent advances in denoising of manifold-valued	
	images	553
	R. Bergmann, F. Laus, J. Persch, and G. Steidl	
	1 Introduction	554
	2 Preliminaries on Riemannian manifolds	557
	2.1 General notation	557
	2.2 Convexity and Hadamard manifolds	560

	3 Intrinsic variational restoration models				
	4 Minimization algorithms				
	4.1 Subgradient descent				
		4.2	Half-quadratic minimization	565	
		4.3	Proximal point and Douglas-Rachford algorithm	567	
	5 Numerical examples 6 Conclusions				
	Re	eferer	nces	574	
15.	ln	nage	and surface registration	579	
	Ke Chen, Lok Ming Lui, and Jan Modersitzki				
	1	Intr	oduction	580	
	2	Mat	hematical background	583	
		2.1	Continuous and discrete images	583	
		2.2	A mathematical framework for image		
			registration	585	
	3	Dist	ance measures	585	
		3.1	Volumetric differences	585	
		3.2	Feature-based differences	588	
	4	Reg	ularization	590	
		4.1	Regularization by ansatz-spaces, parametric	500	
		1 1	registration	590	
		4.2	Quadratic regularizer	591	
		4.5	Registration populties and constraints	592	
		4.4	Registration penalties and constraints Penalties for locally invertible maps	594	
		4.5	Diffeomorphic registration	595	
		4.0	Registration by inverse consistent approach	596	
	5	Surf	ace registration	597	
	5	5 1	Brief introduction to surface geometry	597	
		5.2	Parameterization-based approaches	599	
		5.3	Laplace-Beltrami eigenmap approaches	600	
		5.4	Metric approaches	600	
		5.5	Functional map approaches	601	
		5.6	Relationship between SR and IR	601	
	6	Nun	nerical methods	603	
	7	Dee	p learning-based registration	605	
	8	Con	clusions	606	
	Re	ferer	nces	606	
16.	Metric registration of curves and surfaces using				
	optimal control			613	
	Martin Bauer, Nicolas Charon, and Laurent Younes				
	1 Introduction			614	
	2	Buil	ding metrics via submersions	616	
	3	Opt	imal control framework	619	

	4	Cho	rdal metrics on shapes	622	
		4.1	Motivation	622	
		4.2	General principle	623	
		4.3	Oriented varifold distances	624	
		4.4	Numerical aspects	626	
	5	Intri	nsic metrics	626	
		5.1	Reparametrization-invariant metrics on parametrized		
			shapes	627	
		5.2	The metric on the space of unparametrized shapes	629	
		5.3	The induced geodesic distance	630	
		5.4	The geodesic equation	631	
		5.5	An optimal control formulation of the geodesic problem		
			on the space of unparametrized shapes	632	
		5.6	Numerical aspects	633	
	6	Out	er deformation metric models	635	
	7	A hy	/brid metric model	639	
	8	Con	clusion	641	
	Acknowledgements				
	References		642		
47	r (·	t and a country atmention and any ing		
17.	ET	ficie.	nt and accurate structure preserving	617	
	sc	nem	les for complex nonlinear systems	647	
	Jie Shen				
	1	Intre	oduction	647	
	2	The	SAV approach	649	
		2.1	Suitable energy splitting	653	
		2.2	Adaptive time stepping	654	
	3	Seve	eral extensions of the SAV approach	655	
		3.1	Problems with global constraints	655	
		3.2	L ¹ minimization via hyper regularization	658	
		3.3	Free energies with highly nonlinear terms	659	
		3.4	Coupling with other physical conservation laws	661	
		3.5	Dissipative/conservative systems which are not		
			driven by free energy	664	
	4 Conclusion		666		
	Acknowledgements			666	
	References			666	
				<i>(</i> - -	
Index	(671	