

CONTENTS

Preface

xiii

Part I Introduction

1	Introduction	1
1.1	Multiplicity	2
1.2	Multiple equilibria	3
1.3	Jump Markov processes	4
1.4	Hierarchical state spaces	8
1.5	Scope of modeling	9
2	Simple Illustrative and Motivating Examples	10
2.1	Stochastic Descriptions of Economic Variables	10
2.1.1	Distinguishable and Exchangeable Agents	11
2.2	Entropy: A Measure of Economic Activity	14
2.2.1	Maximizing Entropy	15
2.2.2	Laplace Transform and Moment-Generating Functions	16
2.2.3	Replacing the Sum with the Maximum Term	17
2.3	Empirical Distributions	18
2.3.1	Examples	19
2.3.2	Multiplicity of Microstates	19
2.3.3	Sanov's Theorem	22
2.3.4	Conditional-Limit Theorem	23
2.4	Stochastic Dynamics and Processes	23
2.4.1	Mean First-Passage Times	23
2.4.2	Dynamics with Multiple Equilibria	25
2.4.3	Random Partition of Agents by Types	28
2.4.3.1	Pólya's Urn	28
2.4.3.2	Generalized Pólya Urn Model	28

vii

2.5	Hierarchical State Spaces	29
2.5.1	Examples of Hierarchically Structured State Spaces	32
2.5.1.1	Coin-Tossing Spaces	32
2.5.1.2	K -Level Classification	33
2.5.1.3	Pattern Classification	34
2.5.2	Tree Metric and Martingales	35
2.5.3	Markov Chains on Binary Trees	36
2.5.4	Aggregation of Dynamics on Trees	38
3	Empirical Distributions: Statistical Laws in Macroeconomics	40
3.1	Model Descriptions	40
3.1.1	Micro and Macro Descriptions of Models	40
3.1.2	Multiplicity of Microstates	42
3.2	Entropy and Relative Entropy	45
3.2.1	Kullback–Leibler Divergence Measure	46
3.2.2	Boltzmann and Shannon Entropies	47
3.3	Gibbs Distributions	50
3.3.1	Discrete-Choice Models	50
3.3.2	Detailed Balance and Gibbs Distributions	52
3.3.3	Conditional-Limit Theorems: Gibbs Conditioning Principle	53
3.3.3.1	Example of Asymptotic Independence	55
3.4	Maximizing Equilibrium Probabilities or Minimizing Potential	55
3.4.1	Optimization and Gibbs Distributions	57
3.5	Finite-State Markov Chains	58
3.5.1	Entropy Maximization	59
3.5.2	Cost Minimization with Markov-Chain Dynamics	61
3.6	Large Deviations	62
3.6.1	Example of Asset Returns	63
3.6.2	Chernoff Bounds	64
3.6.3	Tilted Distribution and Lower Chernoff Bound	68
3.6.4	Example of Large-Deviation Analysis	70
3.6.4.1	Sample Mean Greater than the Expected Value	71
3.6.5	Gärtner–Ellis Theorem	72
3.6.5.1	Examples of Dependent Asset Returns	73
3.7	Sanov’s Theorem	73
3.7.1	From Sanov’s Theorem to Cramér’s Theorem	76
3.8	Conditional-Limit Theorem	79

Part II Modeling Interactions

4	Modeling Interactions I: Jump Markov Processes	81
4.1	Market Participation and Other Discrete Adjustment Behavior	81
4.2	Construction and Infinitesimal Parameters	83

4.3	Examples	88
4.3.1	Birth-and-Death Processes	88
4.3.2	Poisson Processes	88
4.4	Equations for Averages: Aggregate Dynamics	89
4.4.1	Example	92
4.5	Multidimensional Birth-and-Death Processes	93
4.5.1	Epidemic Model Interpreted Economically	95
4.5.2	Open Air Market Linear Model I	95
4.5.3	Open Air Market Linear Model II	97
4.5.4	Nonlinear Birth-and-Death Models	99
4.5.5	Birth-and-Death Processes For Partition Patterns	101
4.6	Discrete Adjustment Behavior	102
4.6.1	Example of Employment Adjustment Processes I	104
4.6.2	Example of Employment Adjustment Processes II	107
4.6.3	Example of Inferring Microstate Distribution	108
4.7	Generalizations	109
4.7.1	Hazard Functions: Age-Specific Transition Rates	110
5	Modeling Interactions II: Master Equations and Field Effects	113
5.1	Master Equations	115
5.1.1	A Collection of Independent Agents	118
5.2	Structure of Transition Rates	120
5.3	Approximate Solutions of the Master Equations	121
5.3.1	Power-Series Expansion	122
5.3.2	The Method of Kubo	124
5.4	Macroeconomic Equation	127
5.5	Specifying Transition Rates: Examples	128
5.5.1	Two-Sector Capital Reallocation Dynamics	129
5.5.2	Exchange-Rate Pass-Through	133
5.6	Field Effects: Stochastic Nonlocal and Diffuse Externalities	134
5.7	Generalized Birth-and-Death Models	135
5.7.1	Mean Field Approximation of Transition Rates	137
5.8	Expressing Relative Merits of Alternative Decisions	138
5.9	Equilibrium Probability Distributions	139
5.10	Example of Multiple Equilibria	143
5.10.1	Model	143
5.10.2	Solutions	144
5.10.3	Some Simulation Results	145
5.11	First Passage Times	147
5.11.1	First-Passage Times of Unstable Dynamics	150
5.12	The Master Equation for Hierarchical Dynamics	154
5.13	The Fokker–Planck Equation	157
5.13.1	Power Series Expansion	157

5.13.2	The Kubo Method	160
5.14	The Diffusion-Type Master Equation	163
5.14.1	Ornstein–Uhlenbeck Model	164
5.14.2	Wright–Fisher Model: A Binary-Choice Model	166
5.14.2.1	Unemployment-Rate Model	167
5.14.3	Logistic Model	170
6	Modeling Interactions III: Pairwise and Multiple-Pair Interactions	171
6.1	Pairwise or Multiple-Pair Interactions	171
6.1.1	Ising Model	171
6.1.1.1	Long-Range Pairwise Interactions	172
6.2	A Model of Pairwise Externality	173
6.2.1	Potentials and Equilibrium Probability Distributions	174
6.2.2	Analogy Between Economic Agents and Neurons	176
6.3	Example of Information-Exchange Equilibrium Distribution	177
6.4	Time Evolution of Patterns of Interaction	181
Part III Hierarchical Dynamics and Critical Phenomena		
7	Sluggish Dynamics and Hierarchical State Spaces	184
7.1	Examples of Hierarchically Structured State Spaces	184
7.1.1	Correlated Patterns of Macroeconomic Activity	184
7.1.2	Vectors in an Infinite Dimensional Space	185
7.1.3	Cost Barriers as Ultrametrics	186
7.1.4	Voter Preference Patterns Among Alternatives	187
7.1.5	Champernowne’s Income Distribution Model	188
7.1.6	Random Walks on a Logarithmic Scale	190
7.1.7	Random Multicomponent Cost	191
7.1.7.1	Example of a Two-Level Tree	192
7.1.8	Martingales	194
7.1.9	Branching Processes	195
7.2	Dynamics on Hierarchical State Spaces	197
7.2.1	Ultrametrics: Hierarchical Distance	198
7.2.2	Ogielski–Stein Model	200
7.2.2.1	An Example of Aggregation of Hierarchical Dynamics	203
7.2.2.2	A Nonsymmetric Tree	204
7.2.3	Schreckenberg’s Model	205
7.3	Pruning Trees: Aggregation of Hierarchical Dynamics	207
7.3.1	Renormalization Group Theory	208
7.3.2	Collet–Eckman Model	210
7.3.3	Idiart–Theumann Model	211

8 Self-organizing and Other Critical Phenomena in Economic Models	213
8.1 Sudden Structural Changes	213
8.2 Piling-Up, or Dense Occupancy, of a Common State	215
8.3 Phase Transitions in the Ising Tree Model	219
8.3.1 Phase Transitions in a Random Cost Model	221
8.4 Example of a Two-Level Model	222
8.5 Random Cost Model	224
Elaborations and Future Directions of Research	226
E.1 Zipf's Distribution in Economic Models	227
E.1.1 Bose–Einstein Allocations	229
E.1.1.1 Two-Level Hierarchy	230
E.1.1.2 Three-Level Hierarchical Classification	231
E.1.2 Dirichlet–Multinomial Model of Chen	232
E.1.3 Applications	236
E.2 Residual Fraction Model	238
E.3 Frequency Spectrum	239
E.4 Statistical Distribution of Sizes of Attractive Basins	240
E.5 Transient Distributions	245
Appendix	246
A.1 The Method of Laplace	246
A.1.0.1 Example of Error Integral	247
A.1.0.2 Example of Stirling's Formula	248
A.1.0.3 Example of the Partition-Function Evaluation	248
A.1.1 Rate Functions	249
A.1.1.1 Example of Partition Function Evaluation	250
A.2 First-Passage Times	251
A.2.1 Discrete-Time Markov Chains	251
A.2.2 Example of a Random-Walk Model	252
A.2.3 Continuous-Time Markov Chains	253
A.2.3.1 Simple Random Walks	253
A.2.4 A Standard Wiener Process	254
A.2.5 First-Passage Times to Absorbing Barrier	254
A.3 Exchangeable Processes	255
A.4 Low-frequency Behavior	258
A.5 Lyapunov Functions	261
A.6 Fokker–Planck Equations and Detailed Balance	264
A.6.1 Fokker–Planck Equations and Stochastic Dynamics	264
A.6.2 Detailed Balance and the Fokker–Planck Equation	265
References	268
Index	281