

---

# Contents

Preface	ix
Chapter 1. Introduction	1
§1.1. Resonances in scattering theory	1
§1.2. Semiclassical study of resonances	7
§1.3. Some examples	8
§1.4. Overview	13
<b>Part 1. POTENTIAL SCATTERING</b>	
Chapter 2. Scattering resonances in dimension one	21
§2.1. Outgoing and incoming solutions	22
§2.2. Meromorphic continuation	26
§2.3. Expansions of scattered waves	39
§2.4. Scattering matrix in dimension one	45
§2.5. Asymptotics for the counting function	52
§2.6. Trace and Breit–Wigner formulas	59
§2.7. Complex scaling in one dimension	70
§2.8. Semiclassical study of resonances	82
§2.9. Notes	91
§2.10. Exercises	92
Chapter 3. Scattering resonances in odd dimensions	95
§3.1. Free resolvent in odd dimensions	96
§3.2. Meromorphic continuation	108

§3.3.	Resolvent at zero energy	116
§3.4.	Upper bounds on the number of resonances	125
§3.5.	Complex-valued potentials with no resonances	129
§3.6.	Outgoing solutions and Rellich's theorem	131
§3.7.	The scattering matrix	143
§3.8.	More on distorted plane waves	155
§3.9.	The Birman–Kreĭn trace formula	159
§3.10.	The Melrose trace formula	177
§3.11.	Scattering asymptotics	187
§3.12.	Existence of resonances for real potentials	205
§3.13.	Notes	207
§3.14.	Exercises	210
<b>Part 2. GEOMETRIC SCATTERING</b>		
Chapter 4.	Black box scattering in $\mathbb{R}^n$	217
§4.1.	General assumptions	218
§4.2.	Meromorphic continuation	223
§4.3.	Upper bounds on the number of resonances	235
§4.4.	Plane waves and the scattering matrix	250
§4.5.	Complex scaling	268
§4.6.	Singularities and resonance-free regions	289
§4.7.	Notes	300
§4.8.	Exercises	303
Chapter 5.	Scattering on hyperbolic manifolds	305
§5.1.	Asymptotically hyperbolic manifolds	307
§5.2.	A motivating example	314
§5.3.	The modified Laplacian	317
§5.4.	Phase space dynamics	323
§5.5.	Propagation estimates	332
§5.6.	Meromorphic continuation	341
§5.7.	Applications to general relativity	351
§5.8.	Notes	362
§5.9.	Exercises	364

**Part 3. RESONANCES IN THE SEMICLASSICAL LIMIT**

Chapter 6. Resonance-free regions	371
§6.1. Geometry of trapping	373
§6.2. Resonances in strips	380
§6.3. Normally hyperbolic trapping	392
§6.4. Logarithmic resonance-free regions	403
§6.5. Lower bounds on resonance widths	408
§6.6. Notes	418
§6.7. Exercises	421
Chapter 7. Resonances and trapping	425
§7.1. Lower bounds on the resolvent	426
§7.2. Semiclassical growth estimates	432
§7.3. From quasimodes to resonances	437
§7.4. The Sjöstrand trace formula	446
§7.5. Resonance expansions for strong trapping	456
§7.6. Notes	467
§7.7. Exercises	468

**Part 4. APPENDICES**

Appendix A. Notation	475
§A.1. Basic notation	475
§A.2. Functions	476
§A.3. Spaces of functions	477
§A.4. Operators	477
§A.5. Estimates	478
§A.6. Tempered distributions	479
§A.7. Distributions on manifolds and Schwartz kernels	480
Appendix B. Spectral theory	483
§B.1. Spectral theory of self-adjoint operators	483
§B.2. Functional calculus	488
§B.3. Singular values	489
§B.4. The trace class	492
§B.5. Weyl inequalities and Fredholm determinants	497
§B.6. Lidskiĭ's theorem	504

---

§B.7. Notes	506
§B.8. Exercises	506
Appendix C. Fredholm theory	507
§C.1. Grushin problems	507
§C.2. Fredholm operators	509
§C.3. Meromorphic continuation of operators	513
§C.4. Gohberg–Sigal theory	516
§C.5. Notes	524
§C.6. Exercises	524
Appendix D. Complex analysis	527
§D.1. General facts	527
§D.2. Entire functions	531
Appendix E. Semiclassical analysis	535
§E.1. Pseudodifferential operators	536
§E.2. Wavefront sets and ellipticity	556
§E.3. Semiclassical defect measures	566
§E.4. Propagation estimates	569
§E.5. Hyperbolic estimates	586
§E.6. Notes	603
§E.7. Exercises	604
Bibliography	613
Index	631