

Contents

1	Overview of Book	1
1.1	Introduction.....	1
1.2	Fundamental Issues of a Static Problem	2
1.2.1	Governing Equation with Imperfections.....	2
1.2.2	Simple Examples of Bifurcation Behavior	4
1.3	Overview of Theoretical Concepts	10
1.3.1	Imperfection Sensitivity Law	10
1.3.2	Worst Imperfection of Structural Systems.....	12
1.3.3	Random Variation of Imperfections	14
1.3.4	Experimentally Observed Bifurcation Diagrams.....	15
1.4	Overview of Theoretical Tools	17
1.4.1	Group-Theoretic Bifurcation Theory	17
1.4.2	Block-Diagonalization in Bifurcation Analysis	20
1.5	Overview of Bifurcation of Symmetric Systems	23
1.5.1	Recursive Bifurcation and Mode Switching of Sands	23
1.5.2	Recursive Bifurcation of Steel Specimens	25
1.5.3	Echelon Modes on Uniform Materials.....	27
1.5.4	Flower Patterns on a Honeycomb Structure.....	32
	Summary	32

Part I Imperfect Behavior Around Simple Critical Points

2	Local Behavior Around Simple Critical Points	35
2.1	Introduction.....	35
2.2	General Mathematical Framework	36
2.2.1	Governing Equation with Imperfections.....	36
2.2.2	Critical Point.....	37
2.2.3	Reciprocity.....	39
2.2.4	Stability	40

2.3	Illustrative Example of Bifurcation Analysis	41
2.3.1	Governing Equation	41
2.3.2	Exact Analysis	43
2.3.3	Asymptotic Analysis	45
2.4	Liapunov–Schmidt Reduction	46
2.4.1	Reduction Procedure	46
2.4.2	Criticality Condition	49
2.4.3	Direction of the Bifurcating Path	51
2.4.4	Stability	53
2.4.5	Power Series Expansion of Bifurcation Equation	54
2.5	Classification of Simple Critical Points	56
2.5.1	Limit Point	59
2.5.2	Transcritical Bifurcation Point	60
2.5.3	Pitchfork Bifurcation Point	63
2.6	Example of Pitchfork Bifurcation	68
2.6.1	Exact Analysis	69
2.6.2	Asymptotic Analysis	71
2.7	Appendix: Numerical Bifurcation Analysis Procedure	72
2.7.1	Path Tracing	72
2.7.2	Singularity Detection	74
2.7.3	Branch Switching Analysis	75
2.8	Problems	76
	Summary	76
3	Imperfection Sensitivity Laws	77
3.1	Introduction	77
3.2	Imperfection Sensitivity Laws	79
3.2.1	Limit Point	80
3.2.2	Transcritical Bifurcation Point	81
3.2.3	Pitchfork Bifurcation Point	81
3.2.4	Systematic Derivation	83
3.3	Imperfection Sensitivity of Simple Structures	87
3.3.1	Propped Cantilever	87
3.3.2	Truss Arches	89
3.4	Realistic Example: Elastic–Plastic Plate	93
3.4.1	Ultimate Buckling Strength	95
3.4.2	Imperfection Sensitivity Laws	95
3.5	Appendix: Hilltop Bifurcation of Steel	97
3.6	Problems	99
	Summary	99
4	Worst Imperfection (I)	101
4.1	Introduction	101
4.2	Illustrative Example	102
4.2.1	Governing Equation and Imperfection Sensitivity	102
4.2.2	Worst Imperfection	104

4.3	Theory of Worst Imperfection	107
4.3.1	Formulation	107
4.3.2	Derivation of Worst Imperfection.....	109
4.4	Imperfection with Multiple Categories.....	111
4.5	Worst Imperfection of Simple Structures	113
4.5.1	Truss Arches	113
4.5.2	Regular-Hexagonal Truss Dome	118
4.6	Problems	119
	Summary	120
5	Random Imperfection (I)	121
5.1	Introduction.....	121
5.2	Probability Density Functions of Critical Loads.....	123
5.2.1	Imperfection Coefficient	123
5.2.2	Normalized Critical Load	125
5.2.3	Critical Load	127
5.3	Evaluation of Probability Density Functions	128
5.3.1	Theoretical Evaluation Procedure.....	129
5.3.2	Semi-empirical Evaluation Procedure	129
5.4	Distribution of Minimum Values	130
5.5	Scatter of Critical Loads of Structures and Sands	133
5.5.1	Simple Example	133
5.5.2	Sand Specimens	136
5.5.3	Truss Tower Structure	137
5.6	Problems	140
	Summary	140
6	Experimentally Observed Bifurcation Diagrams	141
6.1	Introduction.....	141
6.2	The Koiter Two-Thirds Power Law	144
6.3	Extensions of the Koiter Law	145
6.3.1	Crossing-Parabola Law.....	145
6.3.2	Laws for Experimentally Observed Bifurcation Diagrams	147
6.4	Recovering the Perfect System from Imperfect Systems.....	152
6.4.1	Recovery from a Single Imperfect Path	152
6.4.2	Recovery from a Series of Imperfect Paths.....	154
6.5	Examples of Observed Bifurcation Diagrams	155
6.5.1	Regular-Hexagonal Truss Dome	155
6.5.2	Sand Specimens	159
6.6	Problems	164
	Summary	164

Part II Theory of Imperfect Bifurcation for Systems with Symmetry

7	Group and Group Representation	167
7.1	Introduction	167
7.2	Group	168
	7.2.1 Basic Concepts	168
	7.2.2 Conjugacy	170
	7.2.3 Direct Product and Semidirect Product	171
7.3	Group Representation	172
	7.3.1 Basic Concepts	172
	7.3.2 Irreducible Representation	177
	7.3.3 Absolute Irreducibility	181
	7.3.4 Schur's Lemma	182
7.4	Block-Diagonalization Under Group Symmetry	184
	7.4.1 An Illustrative Example	184
	7.4.2 Block-Diagonalization Method: Basic Form	186
	7.4.3 Block-Diagonalization Method: Extended Form	194
7.5	Block-Diagonalization of Symmetric Plate Element	196
	7.5.1 Symmetry of Element Stiffness Matrix	197
	7.5.2 Irreducible Representations	198
	7.5.3 Block-Diagonalization	199
7.6	Problems	200
	Summary	200
8	Group-Theoretic Bifurcation Theory	201
8.1	Introduction	201
8.2	Bifurcation Due to Reflection Symmetry	202
8.3	Symmetry of Equations	204
	8.3.1 Group Equivariance of Governing Equation	204
	8.3.2 Equivariance of Linear Parts	206
	8.3.3 Group-Theoretic Critical Point	207
8.4	Liapunov-Schmidt Reduction	208
	8.4.1 Inheritance of Symmetry and Reciprocity	208
	8.4.2 Reduction Procedure	209
	8.4.3 Group Equivariance in the Reduction Process	212
	8.4.4 Criticality Condition	214
	8.4.5 Direction of Bifurcating Paths	215
8.5	Symmetry of Solutions	217
	8.5.1 Ordinary Point	218
	8.5.2 Critical Point	219
	8.5.3 Orbit	221

8.6	Simple Critical Point Under Symmetry	221
8.6.1	Limit Point	222
8.6.2	Pitchfork Bifurcation Point	222
8.7	Equivariant Branching Lemma	223
8.8	Block-Diagonalization of Jacobian and Imperfection Sensitivity Matrices	227
8.9	Example of Symmetric System	228
8.9.1	Symmetry Group and Equivariance	229
8.9.2	Irreducible Representations	231
8.9.3	Symmetry of Critical Eigenvectors	232
8.9.4	Symmetry of Imperfection Sensitivity Matrix	233
8.10	Problems	234
	Summary	235
9	Bifurcation Behavior of D_n-Equivariant Systems	237
9.1	Introduction	237
9.2	Dihedral and Cyclic Groups	238
9.2.1	Definition of Groups	238
9.2.2	Irreducible Representations	241
9.3	Symmetry of Solutions	244
9.3.1	Direct Branches	244
9.3.2	Recursive Bifurcation	248
9.3.3	Bifurcation of Domes	249
9.4	Bifurcation Equations for a Double Critical Point	255
9.4.1	Bifurcation Equations in Complex Variables	255
9.4.2	Equivariance	258
9.4.3	Reciprocity	260
9.5	Perfect Behavior Around a Double Critical Point	260
9.5.1	Bifurcating Branches	261
9.5.2	Stability	264
9.5.3	Summary of Perfect Behavior	267
9.6	Imperfect Behavior Around a Double Critical Point	269
9.6.1	Bifurcation Equations in Polar Coordinates	269
9.6.2	Solution Curves	270
9.6.3	Examples of Solution Curves	272
9.7	Imperfection Sensitivity Laws	275
9.7.1	Case $\hat{n} \geq 5$	278
9.7.2	Case $\hat{n} = 3$	279
9.7.3	Case $\hat{n} = 4$	282
9.8	Experimentally Observed Bifurcation Diagrams	283
9.8.1	Crossing-Line Law	283
9.8.2	Simple Bifurcation Point	285
9.8.3	Double Bifurcation Point	286
9.8.4	Numerical Example: Regular-Pentagonal Truss Dome ...	288
9.8.5	Experimental Example: Cylindrical Sand Specimens	291

9.9	Appendix: Double Bifurcation Point on C_n -Symmetric Path	292
9.10	Problems	294
	Summary	295
10	Worst Imperfection (II)	297
10.1	Introduction	297
10.2	Formulation of Worst Imperfection	299
	10.2.1 Group Equivariance	299
	10.2.2 Imperfection Sensitivity Law	300
	10.2.3 Optimization Problems for Worst Imperfection	301
10.3	Simple Critical Points	302
	10.3.1 Worst Imperfection	302
	10.3.2 Resonance of Symmetry	303
10.4	Double Critical Points	304
	10.4.1 Block-Diagonalization	304
	10.4.2 Worst Imperfection	308
	10.4.3 Resonance of Symmetry	309
10.5	Examples of Worst Imperfection	310
	10.5.1 Truss Tents	310
	10.5.2 Regular-Hexagonal Truss Dome	313
10.6	Problems	315
	Summary	316
11	Random Imperfection (II)	317
11.1	Introduction	317
11.2	Probability Density Function of Critical Loads	318
	11.2.1 Formulation	318
	11.2.2 Derivation of Probability Density Functions	320
	11.2.3 Semi-empirical Evaluation	325
11.3	Distribution of Minimum Values	326
11.4	Examples of Scatter of Critical Loads	327
	11.4.1 Regular-Polygonal Truss Tents and Domes	327
	11.4.2 Pentagonal Truss Dome	329
	11.4.3 Cylindrical Specimens of Sand and Concrete	331
11.5	Problems	333
	Summary	334
12	Numerical Analysis of Symmetric Systems	335
12.1	Introduction	335
12.2	Numerical Bifurcation Analysis of Symmetric Systems	336
	12.2.1 Analysis Procedure	336
	12.2.2 Examples of Numerical Bifurcation Analysis	337
12.3	Revised Scaled-Corrector Method	339
	12.3.1 Original Scaled-Corrector Method	340
	12.3.2 Revised Scaled-Corrector Method	342
	12.3.3 Regular-Hexagonal Truss Dome	344

12.4	Use of Block-Diagonalization in Bifurcation Analysis.....	348
12.4.1	Eigenanalysis Versus Block-Diagonalization	348
12.4.2	Block-Diagonal Form for D_n -Symmetric System	354
12.4.3	Block-Diagonal Form for C_n -Symmetric System.....	355
12.5	Problems	359
	Summary	360
13	Efficient Transformation for Block-Diagonalization	361
13.1	Introduction	361
13.2	Construction of Transformation Matrix: Illustration	363
13.2.1	Regular-Triangular Truss	363
13.2.2	Representation Matrix	364
13.2.3	Local Transformation Matrix	366
13.2.4	Assemblage of Local Transformations	368
13.3	Construction of Transformation Matrix: General Procedure	370
13.3.1	Representation Matrix	370
13.3.2	Local Transformation Matrix	375
13.3.3	Assemblage of Local Transformations	379
13.4	Formulas for Local Transformation Matrices	383
13.5	Appendix: Derivation of Local Transformation Matrices	390
13.5.1	Case $\xi = 0z$	391
13.5.2	Case $\xi = 0xy$	391
13.5.3	Case $\xi = 1Mz$	391
13.5.4	Case $\xi = 1Mxy$	393
13.5.5	Case $\xi = 2z$	395
13.5.6	Case $\xi = 2xy$	397
13.6	Problems	401
	Summary	402

Part III Modeling of Bifurcation Phenomena

14	Bifurcation Behaviors of Cylindrical Soils	405
14.1	Introduction	405
14.2	Groups for Spatial Symmetry	409
14.2.1	Symmetry of Cylindrical Domain	409
14.2.2	Subgroups of $D_{\infty h}$	411
14.2.3	Example of Description of Cylindrical Sand Deformation.....	414
14.3	Experiments on Cylindrical Sand Specimens	414
14.3.1	Recursive Bifurcation Behavior.....	417
14.3.2	Mode Switching Behavior	418
14.3.3	Recovery of Perfect System	422
14.3.4	Application of Crossing-Line Law	423

14.4	Appendix: Derivation of Bifurcation Rules	424
14.4.1	Bifurcation of D_{nh} -Equivariant System	425
14.4.2	Bifurcation of D_{nd} -Equivariant System	431
14.5	Problems	433
	Summary	433
15	Bifurcation of Steel Specimens	435
15.1	Introduction	435
15.2	Symmetry of a Rectangular Parallelepiped Domain	436
15.3	Recursive Bifurcation Rule	438
15.4	Experimental Study	439
15.4.1	Effect of Cross-Sectional Shape	441
15.4.2	Recursive Bifurcation	444
15.5	Computational Study	447
15.6	Problems	448
	Summary	448
16	Echelon-Mode Formation	449
16.1	Introduction	449
16.2	Symmetry Group of Cylindrical Domain	454
16.2.1	Geometrical Symmetry	454
16.2.2	Underlying Translational Symmetry	455
16.3	Subgroups for Patterns with High Spatial Frequencies	457
16.3.1	Diamond Pattern	457
16.3.2	Oblique Stripe Pattern	458
16.3.3	Echelon Mode	459
16.4	Recursive Bifurcation Leading to Echelon Modes	462
16.4.1	Direct Bifurcation	462
16.4.2	Recursive Bifurcation via Oblique Stripe Pattern	462
16.4.3	Physical Scenario for Echelon Mode Formation	463
16.5	Experiment on a Soil Specimen	464
16.5.1	Deformation Patterns: Phenomenological Observation	464
16.5.2	Deformation Patterns: Symmetry	466
16.6	Image Simulations for Stripes on Kaolin	467
16.6.1	Image Simulation Procedure	467
16.6.2	Image Simulation for Kaolin	470
16.7	Patterns on Sand Specimens	476
16.7.1	Experiment and Visualization of Strain Fields	476
16.7.2	Image Simulation	477
16.7.3	Numerical Simulation	480
16.7.4	Three-Dimensional Patterns	481

16.8	Appendix: Derivation of Bifurcation Rules	483
16.8.1	Bifurcation of $O(2) \times O(2)$ -Equivariant System	483
16.8.2	Bifurcation of $OB_{n\bar{n}}^{\pm}$ -Equivariant System.....	491
16.8.3	Bifurcation of $D_{\infty\infty}$ -Equivariant System	499
16.9	Problems	501
	Summary	501
17	Flower Patterns on Honeycomb Structures.....	503
17.1	Introduction.....	503
17.2	Symmetry of Representative Volume Element.....	505
17.3	Bifurcation Rule for Representative Volume Element.....	507
17.3.1	Irreducible Representations for 2×2 Cells	507
17.3.2	Simple Critical Points	508
17.3.3	Double Critical Points	510
17.3.4	Triple Critical Points	510
17.4	Derivation of Bifurcation Equation.....	512
17.5	Solving of Bifurcation Equation.....	515
17.5.1	The Representative Case: $\mu = (3, 3)$	515
17.5.2	Another Case: $\mu = (3, 1)$	521
17.5.3	Other Cases: $\mu = (3, 2)$ and $\mu = (3, 4)$	523
17.5.4	Stability of Bifurcating Branches	523
17.5.5	Analysis by Equivariant Branching Lemma.....	524
17.6	Numerical Analysis of Honeycomb Cellular Solids	525
17.7	Irreducible Representations for $n \times n$ Cells	528
17.7.1	Four-Dimensional Irreducible Representations	528
17.7.2	Six-Dimensional Irreducible Representations.....	529
17.7.3	Twelve-Dimensional Irreducible Representations	530
17.8	Solving of Bifurcation Equation for $n \times n$ Cells	530
17.8.1	Bifurcation Point of Multiplicity 6	531
17.8.2	Bifurcation Point of Multiplicity 12	537
17.9	Problems	546
	Summary	546
A	Answers to Problems	547
	References.....	573
	Index.....	583