CONTENTS

Foreword xv
Preface xvii
Introduction 1
I.1 The Importance of Growth 1
I.2 Empirical Regularities about Economic Growth 5
I.3 A Brief History of Modern Growth Theory 9
1 Growth Models with Exogenous Saving Rates (The Solow–Swan Model) 14
1.1 The Basic Structure 14
1.2 The Neoclassical Model of Solow and Swan 16
1.2.1 The Neoclassical Production Function 16
1.2.2 The Fundamental Dynamic Equation for the Capital Stock 17
1.2.3 The Steady State 19
1.2.4 The Golden Rule of Capital Accumulation and Dynamic Inefficiency 19
1.2.5 Transitional Dynamics 22
1.2.6 Policy Experiments 24
1.2.7 An Example: Cobb–Douglas Technology 25
1.2.8 Absolute and Conditional Convergence 26
1.2.9 Convergence and the Dispersion of Per Capita Income 31
1.2.10 Technological Progress 32
1.2.11 A Quantitative Measure of the Speed of Convergence 36
1.3 Models of Endogenous Growth 38
1.3.1 The *AK* Model 39
1.3.2 Endogenous Growth with Transitional Dynamics 41
1.3.3 Constant-Elasticity-of-Substitution Production Functions 42
1.3.4 The Leontief Production Function and the Harrod–Domar Controversy 46
1.3.5 Growth Models with Poverty Traps 49

Appendix Proofs of Various Propositions 52
Proof That Each Input Is Essential for Production with a Neoclassical Production Function 52
Properties of the Convergence Coefficient in the Solow–Swan Model 53
Proof That Technological Progress Must Be Labor Augmenting 54
Properties of the CES Production Function 55
Problems 56
2 Growth Models with Consumer Optimization (The Ramsey Model) 59
2.1 Households 60
2.1.1 Setup of the Model 60
2.1.2 First-Order Conditions 63
2.2 Firms 67
2.3 Equilibrium 70
2.4 Alternative Environments 71
2.5 The Steady State 72
2.6 Transitional Dynamics 74
2.6.1 The Phase Diagram 74
2.6.2 The Shape of the Stable Arm 76
2.6.3 Behavior of the Saving Rate 77
2.6.4 The Paths of the Capital Stock and Output 79
2.6.5 Speeds of Convergence 80
Appendix 2A Log-Linearization of the Ramsey Model 87
Appendix 2B Behavior of the Saving Rate 89
Appendix 2C Proof That $\gamma_{\hat{k}}$ Declines Monotonically If the Economy Starts from $\hat{k}(0) < \hat{k}^*$ 90
Problems 92
3 The Open Economy, Finite Horizons, and Adjustment Costs 96
3.1 An Open-Economy Version of the Ramsey Model 96
3.1.1 Setup of the Model 96
3.1.2 Behavior of a Small Economy's Capital Stock and Output 98
3.1.3 Behavior of a Small Economy's Consumption and Assets 99
3.1.4 The World Equilibrium 100
3.2 The World Economy with a Constraint on International Credit 101
3.2.1 Setup of a Model with Physical and Human Capital 101
3.2.2 The Closed Economy 102
3.2.3 The Open Economy 103
3.3 Variations in Preference Parameters 108
3.4 Economic Growth in a Model with Finite Horizons 110
3.4.1 Choices in a Model with Finite Horizons 110
3.4.2 The Finite-Horizon Model of a Closed Economy 114
3.4.3 The Finite-Horizon Model of an Open Economy 116

3.5 Adjustment Costs for Investment 119
3.5.1 The Behavior of Firms 119
3.5.2 Equilibrium with a Given Interest Rate 122
3.5.3 Equilibrium for a Closed Economy with a Fixed Saving Rate 125
3.6 Some Conclusions 127
Appendix Overlapping-Generations Models 128
Households 128
Firms 130
Equilibrium 130
The Steady State 131
The Golden Rule and Dynamic Efficiency 133
Dynamics 134
Altruism, Bequests, and Infinite Horizons 135
Problems 137
4 One-Sector Models of Endogenous Growth 140
4.1 The *AK* Model 141
4.1.1 Behavior of Households 141
4.1.2 Behavior of Firms 141
4.1.3 Equilibrium 142
4.1.4 Transitional Dynamics 142
4.1.5 Determinants of the Growth Rate 143
4.2 A One-Sector Model with Physical and Human Capital 144
4.3 Models with Learning-By-Doing and Knowledge Spillovers 146
4.3.1 Technology 146
4.3.2 Equilibrium 148
4.3.3 Pareto Nonoptimality and Policy Implications 149
4.3.4 A Cobb–Douglas Example 150
4.3.5 Scale Effects 151
4.4 Government and Growth 152
4.4.1 The Public-Goods Model of Productive Government Services 152
4.4.2 The Congestion Model of Productive Government Services 158
4.5 Transitional Dynamics in an Endogenous Growth Model 161
4.5.1 A Cobb–Douglas Example 161
4.5.2 A CES Example 164
4.6 Concluding Observations 166
Appendix Conditions for Endogenous Growth in the One-Sector Model 167
Problems 169
5 Two-Sector Models of Endogenous Growth (With Special Attention to the Role of Human Capital) 171
5.1 A One-Sector Model with Physical and Human Capital 172
5.1.1 The Basic Setup 172
5.1.2 The Constraint of Nonnegative Gross Investment 175

5.2 Different Technologies for Production and Education 179
5.2.1 The Model with Two Sectors of Production 179
5.2.2 The Uzawa–Lucas Model 182
5.2.3 The Generalized Uzawa–Lucas Model 196
5.2.4 The Model with Reversed Factor Intensities 197
5.3 Conditions for Endogenous Growth 198
5.4 Summary Observations 200
Appendix 5A Transitional Dynamics with Inequality Restrictions on Gross Investment in the One-Sector Model 201
Appendix 5B Solution of the Uzawa–Lucas Model 204
Appendix 5C The Model with Reversed Factor Intensities 208
Problems 210
6 Technological Change: Models with an Expanding Variety of Products 212
6.1 Models with a Variety of Producer Products 213
6.1.1 Production with a Fixed Number of Products 213
6.1.2 Expansions in the Variety of Products 215
6.1.3 Households and Market Equilibrium 218
6.1.4 Determinants of the Growth Rate 220
6.1.5 Pareto Optimality 220
6.1.6 Erosion of Monopoly Power 223
6.1.7 Romer's Model of Technological Change 226
6.2 Models with a Variety of Consumer Products 231
6.2.1 Varieties of Consumer Goods 231
6.2.2 A Comparison of Consumer Variety with Producer Variety 236
6.3 Concluding Observations 237
Problems 238
7 Technological Change: Models with Improvements in the Quality of Products 240
7.1 Sketch of the Model 241
7.2 Behavior of Firms 242
7.2.1 Levels of Quality in the Production Technology 242
7.2.2 The Incentive to Innovate 246
7.2.3 The Behavior of the Aggregate Quality Index 251
7.2.4 The Market Value of Firms 252
7.3 Households and Market Equilibrium 252
7.4 Innovation by the Leader 254
7.4.1 The Leader as a Monopoly Researcher 255
7.4.2 Research by Outsiders 257
7.5 Pareto Optimality 259
7.6 Summary Observations about Growth 262
Problems 263

8 The Diffusion of Technology 265
8.1 A Leader-Follower Model 266
8.1.1 Behavior of Innovators in the Leading Country 267
8.1.2 Behavior of Imitators in the Follower Country 268
8.1.3 Variations in the Cost of Imitation 272
8.1.4 Empirical Implications for Convergence 274
8.2 Mutual Invention and Imitation 276
8.3 Foreign Investment 276
8.4 Leapfrogging 279
8.5 Summary Observations about Diffusion and Growth 281
Problems 281
9 Labor Supply and Population 285
9.1 Migration in Models of Economic Growth 285
9.1.1 Migration in the Solow–Swan Model 286
9.1.2 Migration in the Ramsey Model 294
9.1.3 The Braun Model of Migration and Growth 300
9.2 Fertility Choice 308
9.2.1 An Overlapping-Generations Setup 309
9.2.2 The Model in Continuous Time 311
9.3 Labor/Leisure Choice 321
Appendix 9A The Form of the Utility Function with Consumption and Work Effort 326
Problems 328
10 Data on Economic Growth, Growth Accounting 330
10.1 Panel Data for Countries 330
10.2 Long-term Data on GDP 332
10.3 Regional Data Sets 341
10.3.1 Data for U.S. States 341
10.3.2 Data for European Regions 342
10.3.3 Data for Canadian Provinces 344
10.3.4 Data for Japanese Prefectures 345
10.4 Growth Accounting 346
10.4.1 General Setup 346
10.4.2 Discrete Time and Variable Shares 347
10.4.3 Measuring Input Shares and the Growth Rates of Inputs 348
10.4.4 Results from Growth Accounting 350
10.4.5 Extensions to Include R&D 351
10.4.6 Limitations of Growth Accounting 352
11 Empirical Analysis of Regional Data Sets 382
11.1 Two Concepts of Convergence 383
11.2 Convergence across the U.S. States 387
11.2.1 β Convergence 387
11.2.2 Measurement Error 392
11.2.3 σ Convergence 392

11.3 Convergence across Japanese Prefectures 393
11.3.1 β Convergence 393
11.3.2 σ Convergence across Prefectures 397
11.4 Convergence across European Regions 398
11.4.1 β Convergence 398
11.4.2 σ Convergence 400
11.5 Migration across the U.S. States 401
11.6 Migration across Japanese Prefectures 404
11.7 Migration across European Regions 407
11.8 Migration and Convergence 410
11.9 Conclusions 413

12 Empirical Analysis of a Cross Section of Countries 414
12.1 Losers and Winners from 1965 to 1985 415
12.2 The Empirical Analysis of Growth Rates 420
12.2.1 Effects from State Variables 421
12.2.2 Control and Environmental Variables 422
12.3 Regression Results for Growth Rates 424
12.3.1 A Basic Regression 424
12.3.2 Tests of Stability of Coefficients 436
12.3.3 Additional Explanatory Variables 436
12.3.4 World Bank Data on GDP 444
12.3.5 Results from a Single Cross Section 445
12.4 Sources of Growth for Slow and Fast Growers 446
12.5 Empirical Analysis of the Investment Ratio 451
12.6 Empirical Analysis of Fertility and Health 452
12.6.1 Results for Fertility 453
12.6.2 Results on Health 454
12.7 Summary and Conclusions about Growth 455

Appendix on Mathematical Methods 462
1.1 Differential Equations 463
1.1.1 Introduction 463
1.1.2 First-Order Ordinary Differential Equations 464
1.1.3 Systems of Linear Ordinary Differential Equations 471
1.2 Static Optimization 491
1.2.1 Unconstrained Maxima 491
1.2.2 Classical Nonlinear Programming: Equality Constraints 492
1.2.3 Inequality Constraints: The Kuhn–Tucker Conditions 494
1.3 Dynamic Optimization in Continuous Time 498
1.3.1 Introduction 498
1.3.2 The Typical Problem 499
1.3.3 Heuristic Derivation of the First-Order Conditions 500
1.3.4 Transversality Conditions 503
1.3.5 The Behavior of the Hamiltonian over Time 503
1.3.6 Sufficient Conditions 503
1.3.7 Infinite Horizons 504
1.3.8 Example: The Neoclassical Growth Model 505

1.3.9 Transversality Conditions in Infinite-Horizon Problems 507
1.3.10 Summary of the Procedure to Find the First-Order Conditions 508
1.3.11 Present-Value and Current-Value Hamiltonians 509
1.3.12 Multiple Variables 510
1.4 Useful Results in Matrix Algebra: Eigenvalues, Eigenvectors, and Diagonalization of Matrices 510
1.5 Useful Results in Calculus 512
1.5.1 Implicit Function Theorem 512
1.5.2 Taylor's Theorem 513
1.5.3 L'Hôpital's Rule 514
1.5.4 Integration by Parts 515
1.5.5 Fundamental Theorem of Calculus 515
1.5.6 Rules of Differentiation of Integrals 516
References 518
Index 529